A Simple Approach to Balancing Government Budgets Over the Business Cycle

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Abstract

Despite the renewed interest in fiscal rules to constrain government deficits and debt, most rules provide no guidelines for reaching fiscal objectives in practice. This note demonstrates how to construct simple and transparent savings-rate rules that could aid policymakers if balancing the budget over the business cycle is a goal.

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1. Introduction

In recent decades there has been a renewed interest in fiscal rules to constrain government deficits and debt. While the European Union's (EU) Stability and Growth Pact is a prime example of this movement, countries such as Australia, Canada, New Zealand, and Switzerland have also enacted formal budget rules (Danninger 2002). Efforts to impose a federal balanced budget rule in the US failed by a single vote in 1997.

Most budget rules, including those in the EU, US states, and Swiss cantons, limit the annual difference between revenues and expenditures. However, countries such as Australia, Switzerland, and the United Kingdom have recently enacted rules that target a balanced budget over the business cycle instead of a fiscal year. And while the effects of budget rules remains an area of active research, balancing government budgets over the business cycle is consistent with Barro's (1979) notion of tax smoothing, may dampen business cycles, and has been shown to be welfare-enhancing relative to annual budget rules (Stockman 2001).

Although balanced budget rules vary from country-to-country, Danninger (2002) notes that, while the rules generally have well-defined objectives, practical guidelines to achieve the objectives tend to be vague or absent altogether. In this note we demonstrate how the estimated parameters from a Markov switching regression may be used to construct simple and transparent "savings-rate rules" that could aid policymakers in balancing the budget over the business cycle. Specifically, we calculate the probability that a revenue cycle of a particular length occurs and the fraction of revenue that must be saved in each expansion period of that cycle to finance the (estimated) revenue shortfall that will arise from that cycle. Repeating this process for all reasonable expansion-contraction combinations, we then form a cumulative density function of savings rates and are able to determine the savings rate that would be needed to hedge *x* percent of all possible revenue cycles.

Because our focus is on developing a simple and formal framework to provide guidance to policymakers, we do not focus on the specific fiscal rules of any given country. Instead, we apply our general model to a sample of countries with and without formal balanced budget rules (US, UK, Canada, Australia, and New Zealand). Researchers interested in a specific country could tailor our approach to that country's unique fiscal goals.

2. Measuring Fiscal Cycles

For a given set of policies, government budget positions will normally be procyclical because of procyclical tax bases and countercyclical social assistance expenditures. Therefore, a theoretical measure of how the budget responds to business cycles would be the government's cyclical surplus or deficit holding tax rates and non-social assistance expenditures constant.

In practice however, modeling the cyclical behavior of a government's cyclical surplus or deficit is problematic because if both revenues and expenditures are exogenously determined, then savings is also pre-determined. For example, consider a given cyclical surplus or deficit series that is constructed holding both tax rates and non-social assistance expenditures constant. Any variation in this series will be generated by the changes in tax bases and social assistance spending that occur over the business cycle. If one determines that saving 1 percent of the cyclical surplus during each expansion period is sufficient to offset the expected cyclical deficit, then a policy change must be made, such as increasing tax rates or reducing spending, to achieve the savings target. Such a policy adjustment would violate the assumptions underlying the cyclical surplus/deficit series.

To simplify matters, we model the cyclical behavior of revenue growth and compute savings-rate rules assuming a constant growth rate in government expenditures. The advantage of this simplification is that our methodology is simple and intuitive. The disadvantage is that our estimates do not account for cyclical changes in spending.[1] Thus, our estimates should be viewed as the *minimum* savings rates that governments could follow if maintaining fiscal balance over the business cycle is a policy goal.

Our methodology, which is based on Wagner and Elder's (2006) application to US state rainy day funds, centers on the notion that the growth rate in revenue and GDP can be characterized by Hamilton's (1989) Markov switching regression model. Hamilton (1989) showed that a univariate, two-regime Markov switching model identified business cycle turning points in US GNP growth that closely matched the official NBER turning points. And while numerous applications and extensions to Hamilton's model have followed, Goodwin (1993) and Li *et al* (2005) find that Hamilton's basic model performs very well in identifying business cycle turning points in most developed economies.

We apply a two-regime (high and low) Markov switching regression with no autoregressive terms to real GDP growth in the U.S., UK, Canada, Australia, and New Zealand. For notational purposes, let g_H^{GDP} and g_L^{GDP} denote the estimated growth rates of the high-growth and low-growth regimes respectively. The unobserved regime variable is assumed to follow a first-order, two-state Markov chain. P_{HH} and P_{LL} denote the transition probabilities of being in the high-growth (low-growth) regime at time t conditional on having been in the high-growth

[1] Using sub-national data from the US, Kusko and Rubin (1993) and Sorenson and Yosha (2001) find that revenues react much more strongly to business cyclical fluctuations than expenditures. Hence, it is reasonable to believe that focusing only on revenue will capture the bulk of the cyclical effects on government budgets.

(low-growth) regime at time *t-1*. Finally, letting ε denote the elasticity of a country's revenue growth with respect to real GDP growth holding tax rates constant, the estimated high- and low-growth rates of revenue, g_H and g_L, are given by $g_i = \varepsilon g_i^{GDP}, i = H, L$. Revenue growth will also have the same transition probabilities as GDP growth.

The use of a revenue elasticity is necessary because, while revenues fluctuate due to business cycle swings, published revenue data also include the effects of discretionary tax rate changes. Applying the model directly to revenue data would therefore capture both business cycle and policy changes. Given that a government's revenue elasticity will vary depending on the tax structure and that most countries tax systems are progressive (implying an elasticity greater than unity), we vary the revenue elasticities over a reasonable range (1.2 to 1.7) to examine the sensitivity of our estimates.[2]

Estimates of each country's Markov switching regression, applied to real quarterly GDP growth, are provided in Table 1. Each model was estimated using Kim and Nelson's (1998) Bayesian Gibbs-sampling approach.[3] The rows labeled $E[t_H]$ and $E[t_L]$ denote the expected duration of a high-growth (low-growth) regime, which may be computed as $E[t_j] = (1 - p_{jj})^{-1}$ for $j = H,L$.

[Table 1 about here]

3. Savings-Rate Rules

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[2] Policymakers should have a reasonable estimate of the government's revenue elasticity. For recent work on revenue variability and the estimation of revenue elasticities see Garrett (forthcoming) and Bruce *et al* (2006).

[3] The joint posterior distributions were simulated using 10,000 replications with an additional 2,000 burn-in replications. The transition probabilities, P_{HH} and P_{LL}, have prior Beta distributions of $\beta(9,1)$ and $\beta(8,2)$, while the prior distributions for g_H^{GDP} and g_L^{GDP} are assumed to be normal with means of 0.50 and –0.25 respectively. Our models were estimated using the computer routines described in Kim and Nelson (1999).

The parameters of the regime-switching regressions provide estimates of the amplitude and duration of the regimes in a revenue cycle. In this section we illustrate how savings-rate rules may be constructed. These "rules" are the fraction of revenue that the government would need to save during each expansion period to hedge a certain percentage of all of the possible expansion-contraction combinations that may occur in a given revenue cycle.

To construct the rules, we calculate the accumulated savings from a revenue expansion lasting t_H periods, compare this savings to the revenue shortfall that would prevail from a recession lasting t_L periods, and solve for the savings rate associated with that specific high-growth, low-growth combination. This process is repeated for all plausible expansion-contraction combinations and a cumulative density function is formed.[4]

Given an initial level of revenue equal to R_0, if revenue grows at a constant rate of g_H during each expansion period, then the level of revenue will be equal to $R_0(1+g_H)^{t_H}$ after t_H expansion periods. If policymakers save a fraction of revenue (s) during each period of a high-growth regime, then the government's accumulated savings, compounding at a rate r, may be written as:

$$R_0 s \sum_{j=1}^{t_H} (1+r)^{t_H - j+1} (1+g_H)^j \ . \qquad [1]$$

If revenue growth shifts to a low-growth regime (growing at a constant rate of g_L) but spending continues to grow at a rate of g_H, the shortfall in the first low-growth period will be the difference between the funds available for spending in the last high-growth period after saving a fraction s, $R_0(1+g_H)^{t_H+1}(1-s)$, and revenue in the first low-growth period, $R_0(1+g_H)^{t_H}(1+g_L)$. Assuming no additional savings in low-growth periods, the revenue shortfall in the first low-

[4] In constructing the savings-rate rules, we assume that government spending grows at a constant rate of g_H (which is the average rate of revenue growth during expansions) during both revenue expansions and revenue contractions.

growth period will be given by $R_0\left[(1+g_H)^{t_H+1}(1-s)-(1+g_H)^{t_H}(1+g_L)\right]$. If the downturn persists into a second period, then revenue will be equal to $R_0(1+g_H)^{t_H}(1+g_L)^2$ and the shortfall in just the second period will be $R_0\left[(1+g_H)^{t_H+2}(1-s)-(1+g_H)^{t_H}(1+g_L)^2\right]$. Thus, the cumulative shortfall following t_L periods of low-growth is the sum of t_L similarly constructed terms and may be expressed as:

$$R_0(1+g_H)^{t_H}\left[(1-s)\sum_{i=1}^{t_L}(1+g_H)^i-\sum_{i=1}^{t_L}(1+g_L)^i\right]. \qquad [2]$$

Since [1] is the government's accumulated savings from an expansion lasting t_H periods and [2] is the government’s revenue shortfall from a recession lasting t_L periods, equating [1] and [2] and solving for s gives the fraction of revenue that must be saved during each of the t_H high-growth periods to accumulate savings equal to the shortfall. This savings rate is given by:

$$s(t_H,t_L)=\frac{(1+g_H)^{t_H}\left[\sum_{i=1}^{t_L}(1+g_H)^i-(1+g_L)^i\right]}{\sum_{j=1}^{t_H}(1+r)^{t_H-j+1}(1+g_H)^j+(1+g_H)^{t_H}\sum_{i=1}^{t_L}(1+g_H)^i}. \qquad [3]$$

The savings rate in [3] applies to a specific (t_H, t_L) combination. However, given that the probability of a particular regime lasting exactly t_j periods is $P_j(t_j)=P_{jj}^{t_j-1}-P_{jj}^{t_j}$ for $j = H, L$, the probability that a high-growth regime lasting exactly t_H periods will be followed by a low-growth regime lasting exactly t_L periods is $P_H(t_H)\times P_L(t_L)$, assuming the regime durations are independent. A cumulative density function of savings rates may then be constructed by varying t_L and t_H from 1/3 to 20 years (in increments of 1/3) and calculating the probability that a revenue cycle of exactly that duration occurs.[5,6] Table 2 reports the “Expected Savings Rate” and “75%

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[5] The savings rates given by [3] were computed using the means of the simulated posterior distributions from the Markov switching regressions (e.g. the point estimates shown in Table 1). An alternative approach would be to use

Savings Rate" for each country with revenue elasticities ranging from 1.2 to 1.7. The "Expected Savings Rate" figures are the means of the corresponding cumulative density function, while the "75% Savings Rate" are the savings rates necessary to weather 75 percent of all the revenue expansion-contraction combinations.

[Table 2 about here]

Given a revenue elasticity of 1.5 and an initial balance between revenue and expenditures, if the UK followed its "Expected Savings Rate" and saved 1.622 percent of revenue during each expansion period, then, on average, their savings will equal zero at the end of a revenue cycle. They will generate insufficient savings during some cycles (because of an unusually short expansion, an unusually long contraction, or both) and an excess of savings during other cycles, but *on average* should be able to weather a revenue shortfall without the need for expenditure reductions or tax increases. To hedge 3 out of every 4 cycles, which implies an excess of savings at the end of a 'typical' revenue cycle, saving 2.135 percent of revenue each expansion period would suffice.

And because the savings rate estimates are a function of both the amplitude and expected duration of expansions and contractions, rates vary noticeably across our sample of countries. For instance, although US has longer expected expansions and shorter expected contractions than the UK, the UK's savings rate estimates are lower than the US (for a given elasticity) because the UK's average revenue cycle has a much smaller amplitude than a typical US revenue cycle.

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draws from the entire simulated posterior distributions in place of the posterior means, which has the added benefit that it becomes trivial to form confidence intervals around any savings rate estimate.

[6] Varying both t_L and t_H from 1/3 to 20 years produces 57,600 possible expansion-contraction combinations. Also, equation [3] assumes that time is an integer value. To increase precision, an equivalent functional form for incremental values of time involves replacing all of the summation terms that appear in those equations. This equivalent functional form is $\sum_{i=1}^{M}(1+g_k)^i = (1+g_k)\left[\frac{1-(1+g_k)^{M-1}}{g_k} + (1+g_k)^{M-1}\right]$, where M denotes the appropriate measure of time (either t_H or t_L) and g_k denotes the appropriate mean growth rate (either g_H or g_L).

4. Conclusion

In this note we have demonstrated how the estimated parameters from a Markov switching regression may be used to construct a simple and transparent savings-rate rule that policymakers could follow if an objective is to maintain fiscal balance over the business cycle. The "rules" show a constant fraction of revenue that policymakers would need to save during periods of revenue expansion to hedge the possible expansion-contraction combinations that may occur with a given level of certainty. And although we presented a very general approach to balancing government budgets over the business cycle, our methodology could easily be modified to coincide more closely with a specific government's fiscal objectives.

Table 1 – Markov Switching Parameter Estimates

	UK	***US***	***Canada***	***Australia***	***New Zealand***
g_H^{GDP}	0.852	1.336	1.327	0.983	1.564
g_L^{GDP}	-0.041	-0.275	-0.310	-0.036	-0.723
p_{HH}	0.907	0.940	0.944	0.936	0.935
p_{LL}	0.774	0.749	0.775	0.750	0.722
σ^2	14.904	0.582	0.634	1.309	1.012
$E[t_H]$	10.75	16.67	17.85	15.63	15.38
$E[t_L]$	4.42	3.98	4.44	4.00	3.60

Notes: Parameter estimates are the mean of the posterior distributions. Models were applied to the growth rate in each country's real GDP. Data on real quarterly GDP for each country were obtained from International Financial Statistics. Sample periods are as follows: UK, US, and Canada 1957:Q2 - 2005:Q3; Australia, 1959:Q4 - 2005:Q3, New Zealand, 1987:Q3 - 2005:Q3.

Table 2 – Savings Rate Estimates

	Revenue Elasticity					
	1.2	*1.3*	*1.4*	*1.5*	*1.6*	*1.7*
UK						
Expected Savings Rate	1.283	1.395	1.508	1.622	1.736	1.852
75% Savings Rate	1.693	1.839	1.987	2.135	2.285	2.433
US						
Expected Savings Rate	1.715	1.871	2.028	2.188	2.350	2.514
75% Savings Rate	2.213	2.412	2.616	2.819	3.022	3.233
Canada						
Expected Savings Rate	2.000	2.183	2.369	2.558	2.749	2.943
75% Savings Rate	2.616	2.853	3.090	3.333	3.580	3.830
Australia						
Expected Savings Rate	1.097	1.194	1.292	1.391	1.491	1.592
75% Savings Rate	1.417	1.540	1.667	1.795	1.921	2.051
New Zealand						
Expected Savings Rate	2.185	2.384	2.586	2.792	3.000	3.211
75% Savings Rate	2.780	3.033	3.291	3.546	3.804	4.067

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