Dynamic Fiscal Policy With Regime-Duration Uncertainty:
The Tax-Cut Case

Erick Elder*

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Abstract

This essay incorporates the uncertainty agents face regarding the duration of the governmental deficit policies into the life-cycle model developed by Auerbach and Kotlikoff (1987). I assume that agents have a probability measure over the possible durations of current deficit policies. They make their savings decisions based upon these expectations, which are updated only after observing the government's action each period. In this setup the transition period is more interesting to policy evaluation as compared to the perfect foresight version of the model. Results suggest that individual expectations and the actual fiscal policy implemented interact to determine the effect deficit policies have on capital formation.

Key Words: life-cycle; government deficits; crowding out.

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*College of Business Administration, University of Arkansas at Little Rock, 2801 S. University Ave., Little Rock, AR 72207–1099, USA (e-mail: emelder@ualr.edu). I would like to thank John Duffy and Dave DeJong for their guidance and suggestions. I would also like to thank Laurence Kotlikoff, Nick Feltovich, and anonymous referees for their helpful comments and suggestions. Any mistakes in this essay are my responsibility.

1 Introduction

In *Dynamic Fiscal Policy*, Auerbach and Kotlikoff (1987) (henceforth AK) develop a benchmark model for large overlapping generation (OG) economies. They use the model to analyze the effects of various tax structures and deficit policies on capital formation and welfare. Their results pertaining to the effects that deficit policies have on capital formation are of particular interest; they show that deficit expansions of any duration eventually lead to the crowding out of capital, but *short*-duration tax cuts can lead to short-term increases in capital formation because there exists a temporary increase in the relative attractiveness of investing in the near term as compared to investing later. These results support the traditional view that governmental deficits decrease capital formation by increasing competition for loanable funds, hence increasing interest rates.

The main shortcoming with the existing literature on this topic is that little attention has been given to the potential impact of uncertainty (regarding the duration of a deficit policy) on long term crowding-out results and possible short term crowding in results. The bulk of the life-cycle literature examines the existence of or convergence to a particular steady state under perfect foresight (*e.g.*, see Wallace 1980, Bullard and Duffy 1993, Bullard and Russell 1994, or Rios-Rull 1994), but in order to study the effects of changes in deficit policies within an uncertain environment, an examination of the equilibrium transition path from the initial steady state to the final steady state is required.[1] However, despite the development by Auerbach and Kotlikoff of an algorithm for conducting transition analysis, and seemingly obvious benefits for policy analysis, uncertainty analysis is rarely conducted.[2] The purpose of this essay is to address this shortcoming. The analysis of AK begins with the specification of a zero-debt, balanced-budget steady state. They then consider the impact of an m-period tax cut; that tax cut, coupled with sustained government consumption, creates a non-zero deficit and initiates an accumulation of government debt. Over the length of the tax cut, government debt is endogenous; following the termination of the tax cut, the tax rate is endogenous and is set to keep the per capita stock of debt constant. AK assume that agents have perfect foresight, and this assumption may account for the lack of interest in the transition following a tax cut. Under perfect foresight, the entire transition period is deterministic and is known by all individuals as soon as the government announces its deficit policy intentions. All individuals know the precise values of all variables in all periods, and decisions made immediately following the announcement of a specific policy are consistent with the implementation of that policy. Expectational errors do not play a role in this formulation. The dynamics are therefore predetermined prior to the implementation of the deficit policy, and no revision of expectations is necessary. I will address this issue by adopting a variation of the AK model in which it is assumed individuals are uncertain as to the duration of announced deficit policies. This uncertainty is exogenously introduced through a probability measure that individuals have over the duration of an announced tax reduction. In this setup individuals' expectational errors concerning future tax rates potentially have significant effects in determining the response of the capital stock to a given policy. Adding this type of uncertainty seems quite natural; while annual budget legislation, and even presidential and congressional philosophies, are commonly known, administrations are subject to change with each election, and the resolution of conflicting philosophies is also unclear *ex ante*.

If future tax rates are not known with certainty, then following an announced tax cut individuals make current and future decisions based on the expected duration of the cut. After observing the government's actions in the first period, individuals update their expectations and revise their current and expected future decisions. Individuals continue to update their expectations until the tax cut has ended, at which time agents know the government will then set taxes to maintain a constant stock of government debt. In this setup, each period during the transition is quite important due to the necessity of observing governmental action each period in order to revise expectations. Not surprisingly, when expectations correspond closely with the implemented policy, the results closely correspond with those obtained by Auerbach and Kotlikoff. However, if large expectational errors are present significant differences can occur in transition results as well as final steady state results. I find that qualitative transition results are most heavily influenced by the *expected* duration of tax cuts, whereas quantitative, final steady state capital levels are primarily influenced by the *actual* durations with some uncertainty influence carried over from the transition.

The remaining sections of this essay are organized as follows. Section 2 explains the model which is used and gives the assumed values of the parameters. Section 3 describes the algorithm employed to solve the model; it also explains how expectations are formed, updated, and enter into the optimal household decisions. Section 4 describes various probability measures which are assumed by individuals, and presents the results of differing deficit policies on the formation of capital, and Section 5 presents some *ex-post* welfare analysis. Section 6 concludes with some final remarks and a discussion of possible extensions.

2 The Model

The general framework of this model is a fifty-five period OG economy similar to that used in AK. Use of a fifty-five-period model is convenient for two reasons: one period in the model corresponds to one year of actual time, and fifty-five periods gives time for substitution effects to be sufficiently strong. An individual belonging to the cohort born in time t lives for fifty-five periods with certainty. Individuals enter the labor force at age one, and may work their entire lives; this corresponds to a scenario in which an individual enters the labor force at age twenty, and lives (and could work) until an average age of seventy four. In any period, there are fifty-five such cohorts which are identical in every aspect except size; each successive generation is $(1+g)$ times larger than its predecessor. Working within an OG framework allows for intercohort heterogeneity, but individuals belonging to the same cohort are assumed to be identical. Individuals born at time t have a zero net asset position at time t, whereas individuals aged $j = 2, 3, ..., n$ come into period t with accumulated assets.

2.1 Household Behavior

An individual born at time t chooses expected consumption and leisure patterns to maximize expected utility, which is given by the following time-separable CES utility function[3]

$$E(t)U = \frac{1}{1-\frac{1}{\gamma}} \sum_{j=0}^{n-1} (1+\delta)^{-j} \left\{ \left[E(t)c_t(t+j)^{1-\frac{1}{\rho}} + E(t)\alpha l_t(t+j)^{1-\frac{1}{\rho}} \right]^{\frac{1}{1-\frac{1}{\rho}}} \right\}^{1-\frac{1}{\gamma}} \quad (1)$$

where $c_t(t+j)$ and $l_t(t+j)$ are the time $t+j$ consumption and leisure of an individual born at time t. Individuals maximize (1) subject to a series of one-period constraints (which can be combined into one lifetime budget constraint given by)

$$
\begin{aligned}
c_t(t) + E(t)\left(\sum_{j=1}^{n-1} c_t(t+j) \prod_{i=1}^{j}\{1 + r(t+i)[1 - \tau(t+i)]\}^{-1}\right) \leq \\
w(t)[1 - l_t(t)]e_1[1 - \tau(t)] + \\
E(t)\left(\sum_{j=1}^{n-1} w_t(t+j)[1 - l_t(t+j)]e_{j+1}[1 - \tau(t+j)] \prod_{i=1}^{j}\{1 + r(t+i)[1 - \tau(t+i)]\}^{-1}\right)
\end{aligned}
\tag{2}
$$

and a time constraint, $0 \leq l_t(t+j) \leq 1$ for $j = 1, 2, ..., n$. The time $t+j$ competitive wage is given as $w(t+j)$, time $t+j$ return to savings is given as $r(t+j)$, and $s_t(t+j)$ is the time $t+j$ savings of an individual born at time t. Savings are held in the form of either capital or government bonds to be explained below. The term $w(t+j)e_{j+1}$ is the effective wage and allows for differences in productivity between workers of different generations.[4] The effective labor units, $e_i's$, are only dependent upon the age of an individual and not on real time. At time t, income is taxed at the proportional tax rate, $\tau(t)$. The expectations operator, $E(t)$, is conditional on the information set $\zeta(t)$. Three expectation regimes are examined later: perfect foresight, and two types of uncertainty. Under perfect foresight, all variables are known at time t. Under uncertainty, $\zeta(t)$ only includes the value of all variables realized up to and including time $t-1$, as well as the entire sequence of effective labor units, e_i.

From the maximization of (1) subject to (2), the optimal consumption and leisure choices are obtained, and are then used to determine aggregate savings for any time t. The optimal consumption and leisure decisions are functions of current and future tax rates which are assumed to be taken as the expected tax rates described later. The solution to the individual's constrained optimization problem given by (1) and (2) gives the individual's optimal consumption and leisure decisions for the current period (derivation shown in AK). In the derivation of the time t optimal quantities the agent must also solve for all expected future quantities and prices since current decisions are influenced by expected future prices. Past decisions indirectly affect current decisions through the determination of an individual's current net asset position. Under perfect foresight, "... each year's current decision will be consistent with previously made plans." (Auerbach and Kotlikoff, 1987, p. 29), but when uncertainty is introduced, this is not generally true. In each period, individuals choose current quantities based on expected future prices, and if these expectations are later revised, the expected future prices may change, making current decisions inconsistent with previously made plans.

An important point to note is the source of uncertainty. In the simulations of Section 4, the government announces a tax cut which is to last a pre-specified length of time, but they are not constrained by this announcement; *i.e.*, if the government announces a tax cut of m periods, the tax cut implemented may last for $q \neq m$ periods. Since individuals do not know future tax rates with certainty, all future individual decisions must be based upon individuals' expectations. Neither do individuals know the government's objective function nor do they engage in any form of strategic interaction with the government. Once individuals observe the actual prices and quantities in a particular period, they can revise their expectations. This revision of expectations can alter expected future consumption, affecting current decisions. Individuals must reoptimize under their revised expectations, while taking past prices and quantities as given.

The parameters α, γ, ρ, and δ are taste parameters, and as with all the parameters of this model are set to match those used in AK. The intratemporal elasticity of substitution, ρ (0.8), measures the responsiveness of the leisure-consumption ratio to changes in the wage and is taken to be approximately the value estimated by Ghez and Becker (1975). The intertemporal elasticity of substitution, γ (0.25), measures the responsiveness of the future-to-current consumption ratio, as well as the future-to-current leisure ratio, to changes in the wage. The value of the intertemporal elasticity lies within ranges estimated by Weber (1970), Grossman and Shiller (1981), Mankiw (1981,1985), and Ghez and Becker (1975). The rate of time preference is given by δ, and is chosen to give reasonable initial steady state results relating to the U.S.'s capital/output ratio. Finally, α is a leisure parameter which scales the amount of labor a prime-aged worker supplies and is taken to be 1.5 which results in prime-aged workers allocating approximately 40 hours per week to work.

2.2 Firm Behavior

Firms operate within a competitive market structure, and combine nondepreciating capital, $K(t)$, and aggregate labor, $L(t)$, to produce net-of-depreciation output, $Y(t)$, using a neoclassical, Cobb-Douglas production function

$$Y_t = AK(t)^{\epsilon}L(t)^{1-\epsilon} \tag{3}$$

where A (.8926576)[5] represents a constant level of technology, and ϵ (0.25) is capital's net of depreciation share of income.[6] The aggregate levels of capital and labor are found as[7]

$$K(t) = K(t-1) + S(t-1) - B(t) \tag{4}$$

$$L(t) = \sum_{i=0}^{n-1}(1+g)^{t-i}[1 - l_{t-i}(t)]e_{i+1} \tag{5}$$

where $l_{t-i}(t)$ is the amount of leisure an individual born at time $t-i$ takes in period t, $S(t)$ is time t aggregate savings, and $B(t)$ is the aggregate stock of one-period government debt. Because markets are assumed to be competitive, the pretax wage and gross interest factor are given by the marginal products of labor and capital respectively:

$$w(t) = (1-\epsilon)AK(t)^{\epsilon}L(t)^{-\epsilon} \tag{6}$$

$$r(t) = \epsilon AK(t)^{\epsilon-1}L(t)^{1-\epsilon} \tag{7}$$

2.3 Government Behavior

The government collects taxes in order to finance its consumption; the model does not take into account transfer payments and government spending on capital formation. Government consumption grows at a constant rate equal to the population growth rate. Taxes are raised on income from wages, capital, and bonds. The government issues one-period debt to finance any deficit, and monetary policy is excluded. The stock of one-period governmental debt, $B(t)$, evolves according to

$$B(t+1) = [1 + r(t)]B(t) + G(t) - \tau(t)Y(t) - \tau(t)r(t)B(t) \tag{8}$$

where $\tau(t)$ is the current proportional tax rate, and $G(t)$ is the current level of government consumption. Just prior to implementing a tax cut, the government announces its policy; hence there are no announcement effects which would occur if the government announced its plans two or more periods prior to the implementation of a policy. During the tax cut regime, the shortfall of government revenues must be financed with the issuance of debt; the stock of debt is endogenous during the tax cut, but once the tax cut has ended, taxes are raised in order to keep the stock of debt per capita constant.

3 Solving the Model

3.1 Formation of Expectations

All individuals have identical expectations, and can solve the model under perfect foresight using the algorithm explained in AK which is briefly explained above.[8] As mentioned earlier, three expectation regimes are examined in the simulations of Section 4.[9] The first is perfect foresight which is the AK version. In the other two expectation regimes, individuals are uncertain as to the duration of the tax cut. Under one of the uncertainty regimes, individuals have flat priors over the possible duration of the tax cut which reflects maximum uncertainty. Under a more realistic regime, individuals have some degree of confidence in the government announcement, but some uncertainty still leads individuals to place a positive probability on the tax cut lasting a different duration than specified by the government (in this situation I will refer to individuals as having informative priors).[10] A tax cut is assumed to last no more than twenty periods.[11]

If individuals do not have perfect foresight, they must base their decisions on expected future tax rates, and they do this using a probability measure over the possible durations of the tax cut.[12] Aggregate savings at time t depends upon $n-1$ past and future tax rates. The previous $n-1$

tax rates have been observed by individuals and therefore are known with certainty. At time t, expected future tax rates, $E(t)\tau$, are given by

$$E(t)\tau = \mathbf{\Gamma} * \mathbf{P} \tag{9}$$

where the $t+j^{th}$ $(j = 1, 2, ..., n-1)$ expected tax rate is given by the $j+1^{th}$ element of $\mathbf{\Gamma} * \mathbf{P}$. The matrix $\mathbf{P}$ is an (n-1)x1 vector given by

$$\mathbf{P} = (p_1 \; p_2 \; ... \; p_{n-1})' \tag{10}$$

where p_k is the individual's expectation a tax cut will last k periods.[13] The (n-1)x(n-1) matrix $\mathbf{\Gamma}$ contains the possible tax rate sequences under perfect foresight

$$\mathbf{\Gamma} = \begin{bmatrix} \tau_{t,1} & \tau_{t,2}, & \cdots & \tau_{t,n-1} \\ \tau_{t+1,1} & \tau_{t+1,2}, & \cdots & \tau_{t+1,n-1} \\ \vdots & \vdots & & \vdots \\ \tau_{t+n-2,1} & \tau_{t+n-2,2}, & \cdots & \tau_{t+n-2,n-1} \end{bmatrix} \tag{11}$$

where $\tau_{t+i,j}$ is the tax rate in period $t+i$ if a tax cut of duration j were observed under perfect foresight.[14] Notice that the perfect foresight solution is achieved if the government announces and implements a tax cut of m periods, and individuals completely believe the announcement, setting $p_m = 1$, and $p_j = 0$ for all $j \neq m$. Each period, individuals observe the current tax rate and update their expectations (specifically, they update $\mathbf{P}$). To illustrate this point, assume the government announces an m-period tax cut, but individuals do not completely believe them, so they form a probability measure such that there exists some $p_i > 0$, $i \neq m$. Therefore, in period t, individual decisions must be based upon the following expected tax rates

$$\begin{aligned} E(t)\tau(t) &= p_1\tau_{t,1} + p_2\tau_{t,2} + \cdots + p_{20}\tau_{t,20} \\ E(t)\tau(t+1) &= p_1\tau_{t+1,1} + p_2\tau_{t+1,2} + \cdots + p_{20}\tau_{t+1,20} \\ &\vdots \end{aligned} \tag{12}$$

Suppose that the tax cut begins in period $t = 1$. Each subsequent period following the tax cut, individuals observe whether or not the tax cut is continuing, and if so then $p_1 = p_2 = \cdots = p_t = 0$ while $p_{t+1}, p_{t+2}, \cdots, p_{20}$ are updated using Bayes' Rule. If the tax cut has not ended in period t, individuals update their expectations and reoptimize taking the previous period's decisions as given. Once individuals observe the termination of the tax cut, they know future tax rates. With this information, individuals are able to solve the model, taking all previous decisions as given, and knowing the government will now set the tax rate in order to balance its budget. Note that equilibrium under this expectations scheme is not identical to the perfect-foresight equilibrium. Prior to the tax cut, individuals correctly forecast future tax rates (in a steady state the tax rate is constant), and after the tax break ends individuals once again correctly forecast future tax rates (by assumption), but during the tax cut individuals base their current (and expected future) decisions upon the expected tax rates which are not equal to those actually observed.

3.2 Initial Steady State

The algorithm employed to solve for the initial steady state is similar to that used in AK. A steady state is the solution to a system of nonlinear equations governing individual, firm, and government behavior as explained above. In a steady state, with expectations formed as explained in the previous section, individuals' expectations of future tax rates are identical to the observed tax rates. Therefore, in solving for the initial steady state, the solution is identical to the perfect-foresight solution because there is no uncertainty in the initial steady state. With the parameters given above the wealth-output ratio of the initial steady state is 3.7 compares well with the wealth-income ratio in the U.S. of 3.5. The bond-income ratio in the initial steady state (= 0) is not close to the observed ratio in the U.S. (approximately = .8), but this is simply due to the assumption of a 0 initial debt level which is taken as such to make the analysis of debt accumulation cleaner. The initial steady state real return to capital is 6.70 percent which is significantly above the observed risk-free ex-post real return in the U.S. of 2.9 percent since 1980. The earnings and consumption profiles assumed in the initial steady state are shown by the figure given in AK (pg. 65). These profiles are fairly realistic with full retirement occurring at age 73 (while at age 64 individuals are supplying less than 20 hours per week and by age 69 that fall to less than 10 hours per week). The consumption profile monotonically rises over the life-cycle. The initial steady state savings rate of 3.7 percent is below the average personal savings-disposable income ratio observed over the last quarter century in the U.S. of 7.2 percent but not far from the 5 percent rate observed in the 1990's. The consumption-output ratio is .81 which is reasonably close to the consumption-NDP ratio observed in the U.S. which has been within the range $(.7, .77)$ for the last quarter century.

3.3 Transition

The algorithm used to solve for the transition between the initial and final steady state is similar to the approach in Altig and Carlstrom (1991). A series of per capita aggregate capital stocks and labor supplies are guessed; these guesses are necessary to find a series of pretax wages and interest rates. At time $t = 1$, the government announces and implements a tax cut. Individuals observe this tax cut and using their prior beliefs form a sequence of expected future tax rates which are then used to determine the expected future decisions of an individual in each cohort alive. Only current decisions are actually observed. In time $t > 1$, individuals observe whether the tax cut is continuing and revise their expectations. Once again, only current decisions are observed. This continues for a maximum of twenty periods. Since this is a one-time policy, individuals know all future tax rates with certainty, as well as the tax rates that correspond with those necessary to maintain a constant stock of debt; they know this information immediately following the termination of the tax cut. The economy is then allowed a maximum of 150 periods to settle down to a new steady state before one is imposed by constraining all prices and per capita quantities to be constant. Aggregated quantities of observed savings and labor supply are compared to the initial guess to determine whether an equilibrium transition path is found; if not, new guesses of per capita aggregate capital stock and labor supply are made and another iteration is performed.

This algorithm is similar to that used to solve for the initial steady state and also similar to the algorithm used by AK to solve for their transition, but far more computationally burdensome than either of these. The reason the solution to the transition is more complicated than the steady state is that it is necessary to solve for each period's prices and quantities simultaneously. It is also much more complicated than the AK transition algorithm because in AK, only individuals alive at the time of the initial tax cut have to reoptimize with accumulated assets while taking previous decisions as given. This is necessary because prior to the government announcement, individuals act as if the initial steady state would continue forever. Here, individuals reoptimize each time they revise their expectations; because their expectations are revised each time their expected future decisions are revised, this affects current decisions.

4 Results

Under each of the expectation regimes described above, three separate deficit policies are examined: one-, five-, and twenty-year tax cuts. Under the policy structure explained above, in which taxes are temporarily cut and then eventually raised beyond their initial steady state level, there is a generational redistribution of income from the younger generations (including those not born yet) to the older generations. Since generations differ there are two possible effects working to influence the response of the capital stock during the transition: an income and substitution effect. If future after-tax income is expected to be sufficiently high, the income effect is strong and individuals may decide to increase current consumption. If instead, the expected cost of current consumption in terms of future consumption is high, they may decide to lower current consumption in favor of future consumption. This substitution effect is determined by the after-tax interest rate, because if individuals consume today they must forgo future consumption in the amount of this rate. The expected strength of these two effects determines the level of savings and consumption for each generation, and therefore the pattern aggregate savings follows during the transition. The actual policy implemented by the government is the major determinant of the debt accumulated during the tax cut, and the interaction of savings and the debt determines the pattern followed by the capital stock.

Although the expected strength of these effects determines the pattern followed by the capital stock during the transition, the final steady state capital stock is lower than that observed in the initial steady state regardless of the expectation regime. Even though some generations alive at the time of the tax cut may increase their current savings because they face lower tax rates, future generations face higher tax rates throughout their lives, thus lowering their budget possibilities and savings levels. It is the savings decisions of these individuals which determine the final steady state capital stock; since the savings of these individuals must be reduced, crowding out is inevitable under the policy scheme described above.

4.1 Perfect Foresight

The perfect-foresight simulations of AK have been replicated here, and the results are given in Tables 1-2.[15] Tables 1-2 and 4-5, and 10-11 report the evolution of variables such as the tax rate, pre-tax wage, pre-tax interest rate, labor per capita, capital stock per capita, output, capital-output ratio, and debt-output ratio. under various expectation schemes and tax-cut durations. The tables report the levels of the variables as observed in the initial steady state, several selected years after the implementation of the tax cut, and the final steady state.

Under perfect foresight, agents know with certainty the duration of the tax cut, and the expected strengths of the competing effects are identical to the actual strengths. Therefore, after the government announcement, expectations never need to be revised, and decisions made at one stage of an individual's life are consistent with those made later in life. Under a one-year tax cut, taxes are soon raised and income changes very little; the substitution effect dominates, as crowding in occurs for several periods. Although generations alive at the time of the tax cut experience a brief increase in after-tax income, consumption of the extra after-tax income is delayed because short-term interest rates are relatively high, and remain so until after the tax cut has ended. Eventually though, individuals face higher tax rates throughout their lives, forcing savings to decrease. Under a twenty-year tax cut, generations alive at the time of the tax cut experience an increase in after-tax income for many years. Comparison of consumption by similar generations in similar periods shows that following a tax cut, individuals experiencing a twenty-year tax cut consume more than those experiencing a five-year cut. From this it is concluded that the income effect is dominant under the twenty-year tax cut. Even though individuals experiencing a twenty-year tax cut save more than those experiencing a one-year cut, crowding in does not occur even in the short run because a larger debt is being accumulated.

4.2 Controlling for the size of the Debt

In order to better understand the relationship between the income and substitution effects, simulations are performed which control for the size of the debt. In simulation A and B, the final accumulated debts are identical, but the size and duration of the tax cut differ. In simulation A, a one-year tax cut is given in which taxes are decreased from 15 percent to 10 percent (the same as the one-year tax cut mentioned above). In simulation B, the tax cut lasts for twenty years with taxes set each year such that one-twentieth of the final accumulated debt of simulation A is accumulated in each of the twenty years. The pattern the capital stock follows (Table 3) is qualitatively similar, but more crowding in occurs under the one-year tax cut. A similar experiment is performed in simulations C and D. In simulation C taxes are cut to 10 percent for twenty years, but in simulation D the same debt is accumulated by only a one-year tax cut. The final steady state capital stocks are similar, but during the transition, the capital stocks follow significantly different patterns. In simulation D, crowding in occurs for one period following the tax break, and then sharply decreases. In simulation C, no crowding in occurs and the crowding out generally occurs rather slowly. Interestingly the final steady state capital stock is almost exclusively dependent upon the size of accumulated debt, whereas during the transition the capital stock is influenced much more heavily by the duration of the tax cut.

In all remaining simulations, tax rates are decreased from an initial level of 15 percent to 10 percent for the duration of the regime change.

4.3 Adaptive Scheme 1

Under the first adaptive scheme, individuals have informative priors which are heavily influenced by the government announcement: individuals assign high probabilities (although less than one) to the announced duration. Let m be the announced duration, and p_m be the probability assigned to this duration. Individuals form the matrix $\mathbf{P}$ by assigning probabilities as follows: $p_m = x, p_{m+1} = p_{m-1} = x/d_1, p_{m-2} = p_{m+2} = x/d_2 ...$[16] such that the sum of all possible probabilities is one. Not surprisingly, when the tax cut actually implemented corresponds with that which was announced, the pattern undergone by the capital stock is similar to that observed under the same duration tax cut and perfect foresight. This is especially true in the case of a one-year tax cut. Individuals not only place the highest probability on a one-period tax cut, but in the second period they observe the tax cut was actually one period long; therefore individuals only need to revise their expectations one time.

A more interesting scenario is when the government announcement does not correspond with the actual policy implemented. Descriptive results summarizing the polar cases are shown in Tables 4 and 5. If the government announces a one-year tax cut and implements a twenty-year tax cut there is actually a significant amount of crowding in which occurs during the transition, in contrast to the immediate crowding out which occurs under a perfect foresight, twenty-year tax cut. This is due to the manner in which individuals revise their expectations. In the first period after the tax cut begins, individuals place highest probability to the tax cut lasting one period (and therefore the substitution effect dominates), but after they observe the tax cut continuing in the second period they then place highest probability on the tax cut lasting two periods. Since expected short-duration tax cuts lead to short-term crowding in, there is a temporary increase in the (per-capita) capital stock during the transition. Qualitatively similar results exist as long as individuals believe the tax cut will last for fewer than ten periods. Tables 6-9 summarize the expectational errors individuals make in forecasting future tax rates and help explain the above results. Recall, that when making decisions in any period t, individuals must forecast all future tax rates. The second column of Table 6 contains the mean error and standard deviation of the errors if the government announces a one-year tax cut and actually implements a twenty-year tax cut. The first row of numbers contains the mean and standard deviation of their forecast errors, for all future tax rates, one period after the tax cut begins. Under this regime, the first period following a tax cut, individuals underestimate the tax rate by an average of 11.4 percent. Likewise, in the second period of the tax cut, individuals are now putting heaviest weight on a two-period tax cut and underestimate future tax rates by an average of 11.2 percent. This underestimation is due to the fact that individuals perceive that the tax cut duration is much shorter than it actually is; the actual tax rates will be significantly higher than expected, and therefore individuals underestimate future tax rates. A reasonable characteristic of this specification is that this underestimation decreases as individuals observe the tax cut persisting so the mean and standard deviation of their errors diminishes.

An important implication of the uncertainty can be observed under this type of scheme. If individuals believe a one-year tax cut will occur, but instead the tax cut lasts for twenty years, less crowding out occurs in the final steady state as compared to a twenty-year tax cut under perfect foresight. This is due to the crowding in which occurred soon after the tax cut was implemented. The increase in capital increases lifetime income which helps offset the higher taxes to be faced later in life. The increased capital stock during the transition increases total tax revenue which decreases the magnitude of the deficit during the tax cut. A lower deficit, *ceteris paribus*, translates into a lower level of debt the government must finance. This ultimately decreases the total amount of capital which is crowded out. Under any twenty-year tax cut, the capital stock per capita falls by about fifty percent and output per capita falls by twenty percent which may seem a bit unrealistic. Noticing that the corresponding debt-output ratios are approximately two accounts for these significant results. Although these results seem dramatic, the final steady state wealth-output ratios are still only slightly above four which are comparable to the ratios observed in the U.S.

Under the opposite scenario in which the government announces a twenty-year tax cut and only implements a one-year tax cut, no crowding in occurs. This result contrasts with the crowding in associated with a one-year tax cut under perfect foresight. In this case, individuals place the highest probability on a twenty period tax cut, and the income effect dominates. By the time individuals realize the tax cut only lasted for one period, they face higher tax rates throughout their lives and immediate crowding out occurs. The final steady state results under this regime seem fairly realistic with output and the capital stock per capita only marginally decreasing. In this case, the expectational errors only occur in the first period, and individuals overestimate future tax rates by an average of 11.1 percent per period (Table 9, column 5).

4.4 Adaptive Scheme 2

The second adaptive scheme assumes that individuals have flat priors over the duration of the tax cut, corresponding to a scenario in which individuals face considerable uncertainty concerning duration (perhaps because the government made no announcement concerning duration). Results are reported in Tables 10 and 11. Even under this scheme, a twenty-year tax cut crowds in capital for nine periods in the transition, and once again ultimately crowds out less capital than does a twenty-year tax cut under perfect foresight. A one-year tax cut crowds in less capital during the transition and ultimately crowds out about the same as compared to a one-year tax cut under perfect foresight. Regardless of the adaptive scheme examined, the policy alone no longer determines the pattern followed by the capital stock; instead, it is the interaction between individuals' expectations and the policy implemented which is the determining factor. An interesting point to note is that under a five- or ten-year tax cut, the mean and standard deviation of the expectational errors actually increase as the tax cut continues. The errors are a function of the manner in which individuals form and revise their expectations. A less fortunate characteristic of the expectations specification and updating routine is seen by noting that if individuals have flat prior and either a five- or ten-year tax cut is implemented then expected future tax rates monotonically diverge from the actual future tax rates. It is also interesting to note a feature of this specification is that under a ten-period tax cut after three periods the mean and standard deviation of errors is greater with flat priors than under one-, five-, or ten-period informative priors. On the other hand though, for a twenty-period tax cut lower mean and standard deviation of errors occurs under flat priors as compared to one-, five-, or ten-period informative priors.

5 Welfare

An important normative question not yet addressed is whether the government should implement any tax cut at all. Assuming a one-time policy in which the government attempts to maximize the welfare of its citizens, is there any case in which the government should cut taxes, or is it necessarily the case that any tax cut does not increase welfare? This welfare analysis is assumed to be performed by the government prior to implementing any policy. Furthermore, it is assumed that the government knows the individuals' maximization problem, but that individuals do not know the government's objective function. Prior to implementing any tax cut, assume that the government attempts to maximize the sum of discounted wealth equivalents of its citizens:

$$V = \sum_{i=0}^{n-1} (1+g)^{-i} WE_{t-i} + \sum_{k=1}^{\chi} \beta^k (1+g)^k WE_{t+k} \tag{13}$$

where the tax cut first occurs in period $t = 1$. The wealth equivalent of generation $t + j$, WE_{t+j} represents the increase (or decrease) in lifetime resources needed under the original tax structure to achieve the level of utility realized by a member of that generation under the current tax structure. If a generation is helped by the implementation of a tax cut then their wealth equivalent is greater than one whereas if a generation realizes a lower level of utility under the tax cut regime then that generation's wealth equivalent is less than one. This is a method of quantifying how much each generation is hurt (or helped) by the implementation of a tax cut.[17]

The government's objective function considers the wealth equivalents of the current and χ future generations, with the wealth equivalents of future generations discounted at a rate of β.[18] It is sometimes postulated that governments are more concerned with the present (and getting re-elected) than they are about future generations, but they need to concern themselves with future generations if they want to stay in power into the indefinite future. The discount factor, β, can be thought of as a measure of the government's concern for future generations with lower discount rates implying less consideration for the future.[19]

The optimal policy for this one-time game is potentially a function of χ, β, and the prior beliefs of individuals (and obviously the welfare function which is specified). The results for two extreme values of χ are reported in Tables 12 and 13: one which would indicate the government is behaving myopically, only concerned with the near future ($\chi = 10$), and another which represents a more forward looking government which is concerned with the welfare of generations into the distant future ($\chi > 150$).[20] A low value of χ does not seem unreasonable given the recent discussion concerning Congressional term limits. On the other hand, higher values of χ are reasonable if one assumes a member of a particular political party wishes for her party stay in power after the termination of that member's tenure. Note the interpretation distinction between χ and β. The parameter χ describes the number of future generations the government is considering, whereas β represents the rate at which the government discounts those considered generations.

The results are reported in Tables 12 and 13. Table 12 reports the sum of discounted wealth equivalents (V) under the regime in which the government only considers 10 future generations (and Table 13 reports the results if the government considers 150 future generations). The first column in each table lists the assumed type of individual expectations. The second column lists selected discount factors. The third column lists the discounted sum of wealth equivalents under the policy of "no tax cut" for comparison with the maximum V. (For reference, the first number in this column would be 65 if the the growth rate were 0 because under a situation in which no tax cut was implemented the wealth equivalent would be 1 for each generation and a total of 65 generations would be considered.) The fourth column reports the maximum V which can be achieved under the possible scenarios, and the fifth column reports the policy which is associated with the maximum V of column four. Interestingly, for high discount rates ($\beta > .90$) the optimal policy is dependent only upon χ and not individual's prior beliefs. If $\chi = 10$, the government's optimal policy always involves cutting taxes for twenty periods. It is interesting to note that if individuals have informative priors the optimal government action is to announce a one-period tax cut and implement a twenty period tax cut. As noted above, in this case individuals will consistently underestimate future tax rates and crowding in will occur; so because of the nature of individuals' expectational errors, the government has an incentive to misrepresent their true intentions. If $\chi = 150$, the policy of no tax cut is optimal for reasonable values of β. This is true for any $\chi > 30$ and is intuitively appealing because the longer a tax cut lasts, the fewer productive resources available to future generations.

Instead of using a constant discount rate, it may be desirable to discount by the inverse of the after-tax interest rate since this is the rate at which the government is able to borrow and lend. In this case, the discounted wealth equivalents of χ future generations is given as

$$V = \sum_{i=0}^{n-1}(1+g)^{-1}WE_{t-i} + \sum_{k=1}^{\chi}(1+g)^{k}WE_{t+k}\prod_{j=1}^{k}\{1+r(t+j)[1-\tau(t+j)]\}^{-1}. \tag{14}$$

If the after-tax interest rate is used in discounting future generations then the optimal policy for the government to follow is not to implement a tax cut regardless of how many or how few future generations are considered and regardless of how individuals form expectations. This result is not surprising because that following a tax cut of almost any duration (and under almost any expectations scheme) the after-tax, time t interest rate is higher that the corresponding after-tax, time t interest rate which would have been observed had no tax cut been implemented. Therefore, future generations are discounted more heavily if a tax cut occurs than if no tax had occurred.

If the constant-discount-rate welfare analysis above is taken literally, one could infer a range for the value of χ based upon the actions of the government.[21] If the government cares about generations into the distant future and does not discount them heavily, then the government would not want to impose an unnecessary burden, in the form of a large public debt, on these future generations. For high discount rates ($\beta > .90$) and the values of χ examined here these welfare results suggest it is unnecessary for the government to know the prior beliefs of individuals when forming their optimal policy. The only information the government needs is χ and β which should be known by the government. The prior beliefs are only important in determining the absolute effects of a given policy.

6 Conclusion and Extensions

This essay has examined the effects that various governmental tax-cut policies and individuals' expectations of tax-cut durations have on capital formation; its findings suggest that uncertainty concerning announced durations can have a significant impact on capital formation. Using results obtained under perfect foresight as a benchmark, it is shown that results obtained under various degrees of uncertainty regarding duration can vary significantly, particularly when expectations concerning duration turn out to be quite wrong *ex post*. For example, when a long tax break (twenty periods) is implemented by the government, results obtained under perfect foresight overstate the resulting level of crowding out relative to the alternative expectation schemes examined here. Although crowding in often occurs in the transition period, and crowding out always occurs in the final steady state, the degree of each varies substantially across expectation schemes. The degree of each effect is dependent upon the actual policy implemented and expectations concerning the duration of the announced tax cut.

The government's welfare-maximizing action is dependent upon the level of individual uncertainty as well as the rate it discounts the welfare of future individuals. One caveat associated with this exercise is that individuals do not take into consideration the government's objective function when forming their expectations or future plans. If individuals considered the government's objective, some announcements may not be credible. Another interesting idea is a repeated game in which individual expectations, as well as government's credibility, were endogenized into the model, and the dynamics of the evolution of individuals' expectations are examined. Furthermore, different policy structures could be examined in which taxes are not cut for a set number of periods and then adjusted to maintain a constant stock of debt, but instead taxes could be set so that there is a high degree of correlation between the present period's tax rate and the next. This type of analysis would have a similar objective as that of Altig and Carlstrom (1991), attempting to explain observed phenomenon with a fifty-five period OG model.

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locate in section 4.1

Table 1: Perfect Foresight; 1-Year Tax Cut

Year	τ	w	r	L	K	Y	$\frac{K}{Y}$	$\frac{Debt}{Y}$
initial steady state	.15	1.000	6.70	19.10	95.10	25.47	3.73	0
1	.10	.991	6.88	19.80	95.09	26.16	3.64	0
2	.152	1.002	6.66	19.03	95.42	25.42	3.75	.05
3	.152	1.002	6.67	19.04	95.35	25.42	3.75	.05
4	.152	1.001	6.67	19.04	95.27	25.42	3.75	.05
5	.153	1.001	6.67	19.04	95.20	25.42	3.75	.05
10	.153	1.000	6.67	19.04	94.90	25.40	3.75	.05
30	.153	.998	6.74	19.07	94.19	25.38	3.71	.05
60	.153	.997	6.74	19.08	93.89	25.37	3.70	.05
90	.153	.997	6.76	19.08	93.87	25.37	3.70	.05
final steady state	.153	.997	6.67	19.08	93.87	25.37	3.70	.05

(τ = proportional tax rate, w = wage, r = net interest rate, L = per capita aggregate labor supply, K = per capita aggregate capital stock, Y = per capita output)

locate in section 4.1

Table 2: Perfect Foresight; 20-Year Tax Cut

Year	τ	w	r	L	K	Y	$\frac{K}{Y}$	$\frac{Debt}{Y}$
initial steady state	.15	1.000	6.70	19.10	95.10	25.47	3.73	0
1	.10	.994	6.82	19.58	95.09	25.95	3.67	0
2	.10	.994	6.82	19.56	95.03	25.93	3.67	.05
3	.10	.994	6.82	19.54	94.94	25.90	3.67	.10
4	.10	.994	6.82	19.53	94.82	25.87	3.67	.15
5	.10	.994	6.82	19.51	94.65	25.84	3.66	.20
10	.10	.990	6.88	19.45	93.37	25.70	3.63	.53
30	.253	.967	7.41	17.50	76.17	22.56	3.37	1.85
60	.300	.882	9.77	17.76	53.46	20.88	2.56	2.00
90	.319	.853	10.78	17.79	46.99	20.25	2.32	2.06
final steady state	.331	.839	11.36	17.80	43.73	19.88	2.19	2.10

locate in section 4.2

Table 3: Capital Stock Paths; Simulations A, B, C, and D

Year	K_A	K_B	K_C	K_D
initial steady state	95.09	95.09	95.09	95.09
1	95.09	95.09	95.09	95.09
2	95.42	95.09	95.03	96.95
3	95.35	95.10	94.94	94.79
4	95.27	95.10	94.82	92.73
5	95.20	95.10	94.65	90.78
10	94.85	95.03	93.04	80.83
30	94.17	94.45	75.00	61.01
60	93.89	93.92	53.06	49.03
90	93.87	93.88	46.87	45.47
final steady state	93.87	93.87	43.73	43.54

K_A = capital stock under a 1-year tax cut in which taxes are cut from 15 percent to 10 percent;
K_B = capital stock under a 20-year tax cut in which tax are set so that $1/20^{th}$ of the above debt is accumulated each year for 20 years;
K_C = capital stock under a 20-year tax cut in which taxes are cut from 15 percent to 10 percent each year;
K_D = capital stock under a 1-year tax cut in which taxes are set so that the accumulation of debt is equal to the accumulation of simulation C

locate in section 4.3, ¶2

Table 4: Informative Priors; Government Announces 1-Year Tax Cut and Implements a 20-Year Tax Cut

Year	τ	w	r	L	K	Y	$\frac{K}{Y}$	$\frac{Debt}{Y}$
initial steady state	.15	1.000	6.70	19.10	95.10	25.47	3.73	0
1	.10	.991	6.89	19.83	95.09	26.20	3.63	0
2	.10	.992	6.86	19.79	95.48	26.20	3.65	.05
3	.10	.993	6.83	19.76	95.81	26.19	3.66	.10
4	.10	.995	6.81	19.72	96.07	26.17	3.67	.15
5	.10	.996	6.79	19.69	96.29	26.15	3.68	.20
10	.10	.998	6.74	19.55	96.50	26.01	3.71	.51
30	.247	.973	7.27	17.52	78.16	22.68	3.45	1.77
60	.291	.888	9.55	17.84	55.29	21.03	2.64	1.91
90	.308	.862	10.45	17.90	49.19	20.49	2.41	1.96
final steady state	.317	.850	10.91	17.90	46.43	20.20	2.30	1.99

locate in section 4.3, ¶4

Table 5: Informative Priors; Government Announces 20-Year Tax Cut and Implements a 1-Year Tax Cut

Year	τ	w	r	L	K	Y	$\frac{K}{Y}$	$\frac{Debt}{Y}$
initial steady state	.15	1.000	6.70	19.10	95.10	25.47	3.73	0
1	.10	.994	6.80	19.52	95.09	25.90	3.67	0
2	.153	1.000	6.69	19.05	94.92	25.41	3.73	.05
3	.153	1.000	6.70	19.05	94.88	25.41	3.73	.05
4	.153	1.000	6.70	19.06	94.83	25.41	3.73	.05
5	.153	1.000	6.70	19.06	94.79	25.41	3.73	.05
10	.153	.999	6.71	19.06	94.60	25.40	3.73	.05
30	.153	.998	6.74	19.07	94.12	25.38	3.71	.05
60	.153	.997	6.76	19.08	93.86	25.37	3.70	.05
90	.153	.997	6.76	19.08	93.84	25.37	3.70	.05
final steady state	.153	.997	6.76	19.08	93.84	25.37	3.70	.05

locate in section 4.3, ¶2

Table 6: Expectational Errors: 20-Period Tax Cut

Period	*I.P./ 1-period*	*I.P./ 5-period*	*I.P./ 10-period*	*I.P./ 20-period*	*Flat Priors*
1	-.114 (.068)	-.105 (.064)	-.086 (.056)	-.014 (.011)	-.077 (.048)
2	-.112 (.067)	-.106 (.064)	-.087 (.055)	-.014 (.011)	-.075 (.047)
3	-.110 (.065)	-.106 (.063)	-.087 (.055)	-.014 (.010)	-.073 (.046)
4	-.107 (.063)	-.106 (.063)	-.088 (.055)	-.014 (.011)	-.071 (.044)
5	-.105 (.061)	-.105 (.062)	-.089 (.055)	-.014 (.010)	-.069 (.043)
6	-.101 (.060)	-.102 (.060)	-.089 (.054)	-.014 (.010)	-.067 (.041)
7	-.098 (.058)	-.098 (.058)	-.089 (.054)	-.014 (.010)	-.064 (.039)
8	-.094 (.055)	-.094 (.056)	-.089 (.053)	-.014 (.010)	-.061 (.037)
9	-.090 (.053)	-.090 (.053)	-.088 (.052)	-.014 (.010)	-.058 (.036)
10	-.085 (.050)	-.085 (.050)	-.087 (.051)	-.015 (.010)	-.055 (.034)
11	-.079 (.047)	-.080 (.047)	-.081 (.048)	-.015 (.010)	-.051 (.031)
12	-.073 (.044)	-.073 (.044)	-.075 (.045)	-.015 (.010)	-.047 (.029)
13	-.066 (.040)	-.067 (.040)	-.068 (.041)	-.015 (.010)	-.043 (.027)
14	-.058 (.036)	-.058 (.036)	-.060 (.037)	-.014 (.010)	-.038 (.024)
15	-.049 (.031)	-.049 (.032)	-.051 (.032)	-.014 (.010)	-.032 (.021)
16	-.039 (.026)	-.039 (.026)	-.041 (.027)	-.013 (.009)	-.026 (.018)
17	-.028 (.021)	-.028 (.021)	-.030 (.021)	-.012 (.008)	-.020 (.015)
18	-.016 (.015)	-.016 (.015)	-.018 (.015)	-.009 (.007)	-.012 (.011)
19	-.003 (.008)	-.003 (.008)	-.005 (.008)	-.005 (.005)	-.004 (.006)
20	.011 (.003)	.010 (.003)	.008 (.002)	.001 (.001)	.006 (.002)

I.P./1-period = Individuals have informative priors and expect a one-year tax cut

locate in section 4.3

Table 7: Expectational Errors: 10-Period Tax Cut

Period	*I.P./ 1-period*	*I.P./ 5-period*	*I.P./ 10-period*	*I.P./ 20-period*	*Flat Priors*
1	-.029 (.020)	-.019 (.015)	.001 (.004)	.079 (.050)	.012 (.012)
2	-.026 (.019)	-.019 (.015)	.002 (.004)	.080 (.050)	.014 (.012)
3	-.023 (.017)	-.019 (.015)	.002 (.004)	.080 (.050)	.017 (.013)
4	-.020 (.015)	-.018 (.014)	.002 (.004)	.081 (.050)	.020 (.014)
5	-.017 (.013)	-.017 (0137)	.002 (.004)	.081 (.050)	.022 (.015)
6	-.013 (.011)	-.013 (.011)	.001 (.004)	.082 (.049)	.026 (.016)
7	-.009 (.009)	-.009 (.009)	.002 (.004)	.082 (.049)	.029 (.018)
8	-.004 (.006)	-.004 (.006)	.002 (.004)	.083 (.049)	.032 (.020)
9	.001 (.004)	-.001 (.004)	.004 (.004)	.0983 (.048)	.036 (.022)
10	.007 (.005)	-.006 (.005)	.006 (.005)	.084 (.047)	.040 (.023)

locate in section 4.3

Table 8: Expectational Errors: 5-Period Tax Cut

Period	*I.P./ 1-period*	*I.P./ 5-period*	*I.P./ 10-period*	*I.P./ 20-period*	*Flat Priors*
1	-.009 (.008)	.001 (.003)	.021 (.015)	.100 (.060)	.032 (.020)
2	-.006 (.007)	.001 (.003)	.021 (.015)	.100 (.060)	.035 (.021)
3	-.003 (.005)	.002 (.003)	.021 (.015)	.101 (.059)	.037 (.022)
4	.001 (.003)	.002 (.003)	.022 (.015)	.102 (.059)	.040 (.024)
5	.003 (.004)	.004 (.004)	.022 (.015)	.102 (.059)	.043 (.025)

locate in section 4.3

Table 9: Expectational Errors: 1-Period Tax Cut

Period	*I.P. 1-period*	*I.P. 5-period*	*I.P. 10-period*	*I.P. 20-period*	*Flat Priors*
1	.003 (.002)	.012 (.001)	.033 (.021)	.111 (.064)	.044 (.025)

Table 10: Flat Priors; 1-Year Tax Cut

locate in section 4.4

Year	τ	w	r	L	K	Y	$\frac{K}{Y}$	$\frac{Debt}{Y}$
initial steady state	.15	1.000	6.70	19.10	95.10	25.47	3.73	0
1	.10	.993	6.84	19.64	95.09	26.01	3.66	0
2	.153	1.001	6.68	19.05	95.14	25.42	3.74	.05
3	.153	1.001	6.68	19.05	95.08	25.41	3.74	.05
4	.153	1.001	6.69	19.05	95.03	25.41	3.74	.05
5	.153	1.000	6.69	19.05	94.97	25.41	3.74	.05
10	.153	1.000	6.70	19.06	94.74	25.40	3.73	.05
30	.153	.998	6.74	19.07	94.16	25.38	3.71	.05
60	.153	.997	6.76	19.08	93.88	25.37	3.70	.05
90	.153	.997	6.76	19.08	93.86	25.37	3.70	.05
final steady state	.153	.997	6.77	19.08	93.85	25.37	3.70	.05

locate in section 4.4

Table 11: Flat Priors; 20-Year Tax Cut

Year	τ	w	r	L	K	Y	$\frac{K}{Y}$	$\frac{Debt}{Y}$
initial steady state	.15	1.000	6.70	19.10	95.10	25.47	3.73	0
1	.10	.992	6.85	19.70	95.09	26.08	3.65	0
2	.10	.993	6.84	19.68	95.25	26.07	3.65	.05
3	.10	.994	6.83	19.65	95.37	26.05	3.66	.10
4	.10	.994	6.82	19.63	95.44	26.03	3.67	.15
5	.10	.995	6.81	19.60	95.47	26.02	3.67	.20
10	.10	.995	6.81	19.51	95.02	25.89	3.67	.52
30	.250	.971	7.32	17.51	77.34	22.62	3.43	1.81
60	.295	.886	9.64	17.81	54.49	20.93	2.61	1.95
90	.313	.858	10.59	17.85	48.22	20.36	2.38	2.01
final steady state	.323	.845	11.10	17.85	45.25	20.03	2.27	2.04

locate in section 5, ¶4

Table 12: What Should the Government Do? ($\chi = 10$)

$$V = \sum_{i=0}^{n-1} (1+g)^{-i} WE_{s-i} + \sum_{k=1}^{10} \beta^k (1+g)^k WE_{s+k}$$

Expectations	β	V_{SS}	V_{max}	*Policy to Achieve* V_{max}
Perfect Foresight	1.00	53.23	53.33	implement 20-year tax cut ($\beta \leq 1.00$)
	0.95	50.68	50.82	
	0.90	48.80	48.96	
	0.85	47.41	47.58	
	0.80	46.37	46.55	
	0.75	45.59	45.78	
	0.71	45.10	45.30	
Informative Priors	1.00	53.23	53.35	announce 1-year, implement 20-year tax cut ($1.00 \leq \beta \leq .76$)
	0.95	50.68	50.83	
	0.90	48.80	48.97	
	0.85	47.41	47.59	
	0.80	46.37	46.56	
	0.75	45.59	45.78	implement 20-year tax cut ($.75 \leq \beta$)
	0.71	45.10	45.30	
Flat Priors	1.00	53.23	53.34	implement 20-year tax cut ($\beta \leq 1.00$)
	0.95	50.68	50.83	
	0.90	48.80	48.96	
	0.85	47.41	47.58	
	0.80	46.37	46.55	
	0.75	45.59	45.78	
	0.71	45.10	45.30	

V_{SS} = discounted sum of wealth equivalents if no tax cut is implemented, V_{max} = maximum discounted sum of wealth equivalents (achieved if policy is last column is followed).

locate in section 5, ¶4

Table 13: What Should the Government Do? ($\chi = 150$)

$$V = \sum_{i=0}^{n-1}(1+g)^{-i}WE_{s-i} + \sum_{k=1}^{150}\beta^k(1+g)^kWE_{s+k}$$

Expectations	β	V_{SS}	V_{max}	*Policy to Achieve* V_{max}
Perfect Foresight	1.00	390.9	390.9	do not implement any tax cut ($1.00 \geq \beta \geq 0.89$)
	0.95	66.21	66.21	
	0.90	52.56	52.56	
	0.85	48.64	48.75	implement 20-year tax cut ($\beta \leq 0.88$)
	0.80	46.78	46.94	
	0.75	45.69	45.88	
	0.71	45.10	45.30	
Informative Priors	1.00	390.9	390.9	do not implement any tax cut ($1.00 \geq \beta \geq 0.90$)
	0.95	66.21	50.21	
	0.90	52.56	52.56	
	0.85	48.64	48.76	announce 1-year, implement 20-year tax cut ($0.89 \geq \beta \geq 0.74$)
	0.80	46.78	46.95	
	0.75	45.69	45.88	
	0.71	45.10	45.30	announce and implement 20-year tax cut ($\beta \leq 0.73$)
Flat Priors	1.00	390.9	390.9	do not implement any tax cut ($1.00 \geq \beta \geq 0.90$)
	0.95	66.21	66.21	
	0.90	52.56	52.56	
	0.85	48.64	48.76	implement 20-year tax cut ($\beta \leq 0.89$)
	0.80	46.78	46.95	
	0.75	45.69	45.88	
	0.71	45.10	45.30	

Endnotes

1. Here, differences between initial and final steady states arise from changes in tax rates.
2. See Altig and Carlstrom (1991) for one example of a paper which stresses the importance of the transition.
3. Following the conventional notation, subscripts record birthdates while parenthesis refer to real time. Variables such as wages and net interest rates are given to the household and do not require birthdate subscripts since they are only real time dependent.
4. The effective units pattern is identical to the pattern used in AK and is based upon the estimates of Welch (1979). The effective labor unit associated with an individual of age i is given by $e_i = exp^{4.47+.033i-.00067i^2}$ for $i = 1, 2, ...55$.
5. AK explained that the choice for the parameter A is dependent upon the units in which output is measured, and the particular value is chosen for convenience by making the initial steady state wage of a one-year old equal to one. As was pointed out by a helpful referee though, since the preferences are not consistent with long-term growth the choice of A could affect the results. Therefore, choosing a value for A is not a true normalization. The choice of A does significantly affect the initial steady state capital-labor ratio and the particular A chosen yields a value for this ratio of 5. Choosing lower values of A ($A = .2$) causes this ratio to fall below one while higher values ($A = 1.2$) causes this ratio to rise to 8. In 1990, this ratio was approximately 4.7 so this choice of A should be considered part of the calibration and not a normalization.
6. It can be shown that assuming $\epsilon = .25$ in conjuction with an assumption of no depreciation is approximately equivalent to assuming a capital's gross share of income of .35 with a depreciation rate of 5%. To maintain comparability with AK, the former set of assumptions is retained.
7. Equation (4) is derived from rewriting the market clearing condition $S(t-1) = I(t) + B(t)$ where aggregate investment, $I(t)$, represents the change in the capital stock, $K(t) - K(t-1)$. This condition states that aggregate savings can be used to form new capital or finance the government's debt.
8. A more complete explanation of the algorithm is contained is Section 4.
9. This paper does not attempt to endogenize these expectation, but instead examines the question of what impact does imperfect foresight have on the crowding-out effect of various deficit policies.
10. When individuals have informative priors, they simply weight the government announcement heavily and does not necessitate their priors being correct.
11. The assumption that a tax break can last for a maximum of twenty years maintains comparability with AK, and does not seem likely to significantly alter the results because the probability that individuals assign to a tax cut of longer duration should be small. Within the framework of this model, a perpetual tax cut is impossible to finance.

12. The estimation of this probability measure is beyond the scope of this paper, but one possibility would involve estimating a switching process and converting the resulting probabilities into just such a probability measure.

13. Note: $p_{21} = p_{22} = \cdots = p_{54} = 0$ and $\sum_i p_i = 1$.

14. Note that the Γ matrix contains perfect-foresight tax sequences for tax cuts of different durations; no updating of the Γ matrix occurs. Each of these sequences are based upon a specific accumulation of debt. Since individuals base their decisions upon expected future tax rates, this alters future expected debt which in turn alters future expected tax rates. Simulations were performed in which these tax sequences were updated, and the results were not significantly different than those reported later in the paper. This type of simulation takes an enormous amount of computer time so the simulations which are examined later do not revise the Γ matrix.

15. The results are subject to slight rounding errors.

16. Different sequences for the d_i terms are studied, but for the results reported d_i is determined by $d_i = 2^i$.

17. Note that this essay does not use the Lump Sum Redistribution Authority (LSRA) as is used in AK. The LSRA is a self-financing institution which uses a series of transfer payments to leave the utility of generations alive at the time of the tax cut at the level which would have been realized under the original tax regime, and raise (or lower) the utility of all future generations to some uniform level. The transfer is accomplished the first period of the tax regime change for those individuals alive at the time of the tax cut, and in the first period of life for those individuals not yet born. This structure would not be appropriate within the context of this essay because as soon as any transfer is made to (or taken from) any individual then that individual would be able to infer with certainty the actual length of the tax cut therefore negating any uncertainty.

18. Discount rates in the interval $[0.7, 1.0]$ are examined. This seems like a sensible range in that it does not seem plausible that the government would value future generations more than current ones ($\beta > 1$), or for the government to discount future generations at a rate less than the lower bound above (keeping in mind that the next generation will be able to vote in the next period).

19. As a helpful referee noted, any discounting of future generations maybe considered, in Ramsey's words, "morally reprehensible". Myopic behavior on the part of the government may be represented by the discounting future generations or by only considering χ future generations. Here, I refer to the consideration of only χ future generations as representing this type of behavior.

20. Values of χ in the interval $[10, 150]$ (in increments of 10) are examined, but the results for $\chi > 30$ are all very similar.

21. For example, if a tax cut is ever given then the government must be considering less than 30 future generations.