

Implications of Social Security Reform on Interest Rates: Theory and Evidence

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Abstract

This paper investigates the potential effect that Social Security reform may have on bond and equity returns. We specifically focus on the effect of proposals to shift a portion of the investment of the Social Security Trust Fund to the equities market. Models are developed to demonstrate the relationship between returns and both the relative size of the Social Security Trust Fund and the portfolio allocation of the Trust Fund. Using these two models, we then show that interest rates will increase from either a decrease in the size of the Social Security Trust Fund or a shifting in the investment mix from bonds to equities. We then derive an adjustment factor that relates the magnitude of change in interest rates from either source. We use this adjustment factor in conjunction with estimates of the relationship between government debt and interest rates to forecast both the potential effect on interest rates from shifting part of the Trust Fund to the equity market. Our estimates suggest that investing some of the Social Security funds in equities is not a painless cure-all for the Social Security system and may have some adverse effects in terms of income transfers from American taxpayers to foreign bondholders.

Key words: Social Security Reform; General Equilibrium; Portfolio Choice; Asset Pricing.
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1. Introduction

The Social Security Administration has stated that the current Social Security system will not have sufficient funds to provide the projected level of benefits over the next seventy-five years. The leading edge of the baby-boom generation turns 65 in 2011; two years later the payment of benefits is projected to exceed tax income. By 2024 the total payment of benefits will exceed the tax income plus the earnings from the Trust Fund, which will cause the Trust Fund to decrease in size. By 2037, the OASDI Trust Fund will be depleted, and future benefits must be decreased by twenty five percent to maintain a balance between tax income and benefit payments. In an attempt to maintain a funded Social Security system, several reforms have been proposed. All of these reforms have a number of components, but almost all include some combination that would cut benefits (by increasing “normal” retirement age, decreasing COLA's, or increasing the penalty for early retirement), increase taxes (by increasing the base payroll tax, increasing the income limit, or impose a “transitional tax”), or introduce some degree of privatization into the system (by giving individuals some control over how their contributions are invested or shifting Trust Funds into the equities market¹. This paper focuses primarily on shifting a portion of the investment of the Trust Fund into the equities market. Currently, the trust fund holds only special U.S. Treasury securities, which yield a relatively low risk-free return. Supposedly, the additional expected return from equities (as compared to U.S. Treasury securities) would then avoid an increase in taxes or decrease in benefits. We demonstrate that this is inaccurate because of the change in interest rates resulting from a shift in the investment of the Social Security Trust Fund. Finally, our results focus on the fact that an informed discussion of Social Security reform needs

¹ Although classifying a shift of Trust Funds into the equities market as “privatization” is a little imprecise in terminology, for the purposes of this research shifting funds to the equities market is qualitatively similar to a real privatization. For more details on the various plans, see the *Report of the 1994-1996 Advisory Council on Social Security* (1997).

to explicitly take into account the secondary effects of reform - specifically the effects on interest rates, income tax changes, and income redistribution.

As Diamond (1998) has pointed out, the major proposals for reform differ in three areas: (1) defined benefit or defined contribution plan, (2) how the funds are invested, and (3) the amount of funding. The analysis then generally considers the effects of reform on welfare, national savings, labor-supply decisions, capital formation, risk-sharing, and other economic activity. One area that has not been thoroughly explored is the effect of reform on asset returns. Often, it is assumed that the magnitude of such an effect will be small. However, two assumptions must be true concerning the reform of the Social Security system if interest rates and equity returns are not affected: (1) individuals must take into consideration the investment allocation of Social Security when making their own allocation decision and (2) no individuals can be constrained in terms of their desired allocation, *i.e.*, every individual should be able to invest their entire retirement account (including Social Security) in bonds and/or equities according to their preference. Serious questions as to the validity of these assumptions exist. First of all, under the current defined benefit system, individuals should not care about the investment of Social Security funds because the benefits are fixed regardless of the allocation decision² Secondly, Geanakoplos, et al (1998, page 154) point out that if households are constrained in their investment choices, there would be macroeconomic consequences in terms of changes in asset returns. This could occur if some individuals want all their retirement funds in equities (extreme risk takers) or in bonds (highly risk averse investors) or if some individuals do not have access to credit markets.³ If either of these two assumptions is violated, then asset returns will be affected to some degree by Social Security reform, and specifically by shifting funds to the equities market. The intuition behind this effect on interest rates and asset returns is that if the two

²This view could be modified somewhat if individuals believe that the ability of Social Security to pay the defined benefits is a function of the Social Security allocation decision, *i.e.*, shifting funds to the equities market makes it more likely that benefits will be paid.

above assumptions are violated then a shift in funds to equities is equivalent to a change in preferences, which will change relative prices. In our view, these two assumptions are likely to be violated, and this forms the basis for the model in this paper.

One purpose of this paper is to model the relationship between interest rates and both the size and the investment decision of the Social Security Trust Fund. Specifically, the reform we examine is a forty percent shift of Social Security Trust Funds to the equities market. Our model uses the optimal-portfolio selection rule derived by Merton (1969) in continuous time and Samuelson (1969) within a discrete time framework. Merton (1969) derives the optimal-portfolio selection rule and optimal consumption rule for a given set of expected asset returns. We extend this analysis to a general equilibrium framework to endogenously determine these returns. Using this setup, we first show the effect on interest rates from a shift in the investment of Social Security funds to the equities market, keeping the size of the Trust Fund constant. We demonstrate that interest rates rise as the Social Security Trust Fund investment is shifted to the equities market because the government must now entice more individuals to hold its debt. We then develop a similar model to examine the effect on returns of an increase in the size of the Social Security Trust Fund, keeping the allocation decision of the Trust Fund constant. We show that interest rates should fall as the size of the fund increases. A comparison of the two models shows that an adjustment factor can be used to translate the effect on interest rates of changing the size of Social Security Trust Fund into an equivalent effect for shifting funds to the equities market.

The main contribution of this paper is to indirectly estimate the impact on interest rates from the two types of Social Security Reform: a reallocation of the Trust Fund into equities. This impact is of immense importance when discussing potential reform because it significantly alters the merits of the potential reform plans. Unfortunately, there is no direct historical evidence to

³ In a related paper, Elder and Holland (2000) review the issue of investor constraint in greater detail and find a change in interest rates and equity returns when individuals are constrained.

suggest the magnitude of the effect these types of reforms may have on asset returns. Estimation of the magnitude of the effects is accomplished by using an adjustment factor in conjunction with an estimate of the relationship between government debt and interest rates. The literature on the relationship between government debt and interest rates is widely diversified with respect to both methodology and results, and it is not the purpose of this paper to resolve this specific question. Instead, we examine a variety of specifications that result in a wide range of estimates of the impact of government debt on interest rates. These estimates allow us to define the potential magnitude of the effects of shifting investment to equities. We first estimate the range of the potential impact on interest rates if forty percent of the current Social Security Fund were invested in the equities market. We show that interest rates on government debt could rise even without a change in the national savings rate, labor employment, or capital productivity.

The topic of Social Security reform has attracted a large literature and has been the topic of numerous conferences (Federal Reserve Bank of Boston [1997], American Economic Association [1996], National Academy of Social Insurance [1998], and the recent White House Conference in December 1998). Most studies of Social Security use the life-cycle framework developed by Auerbach and Kotlikoff (1987). This is the basic framework we use, allowing for changes in the savings rate and income taxes, but not the labor/leisure decision.

One important point to note is that a fully dynamic model is beyond the scope of this paper. A fully dynamic model would include features such as the impact of foreign capital flows, how the changes in interest rates and equity returns affect the firms investment and financing decisions, the long-run impact in terms of capital accumulation and growth from changes in investment decisions, the monetary authorities response to changing interest rates, any politics that may influence the government's choice of which equities to invest in, etc. Since many of these issues are already included in current discussions of the merits of the proposed reforms, we choose not to incorporate them in order to maintain focus on two specific issues: the mechanism through which reform affects interest rates (*i.e.*, the impact of reallocating a captured demand

from bonds to equities), and the estimation of the potential change in interest rates from the two different proposed reforms. Our bottom line is that there are no free lunches with respect to the reform of Social Security.

The rest of this paper is organized as follows. Section 2 describes the basic model framework and examines the effect on returns of shifting existing Social Security funds to the equities market, and Section 3 examines the effect on returns of changing the size of the Social Security Trust Fund. Section 4 makes a qualitative and quantitative comparison of the effects on returns between the two models. Then, Section 5 defines the data and empirical methods used to determine a historical relationship between the relative size of the Social Security Trust Fund and interest rates as well as describing the results. Other implications of our research are discussed in Section 6. Finally, a summary of the results and conclusions is presented in Section 7.

2. Shifting Social Security Funds to the Equities Market

This first model demonstrates the effect of shifting a portion of the investment of the Social Security Trust Fund into equities. The model in this section assumes that individuals do not take into account the allocation decisions of the Social Security Trust Fund when making their own allocation decisions except to respond to changes in equilibrium asset returns. In effect, the Social Security Trust Fund is treated as a very large investor. A shift in investing a portion of the Social Security Trust Fund in equities is then equivalent to a change in preferences. Following Merton (1969), suppose that there is one risk-free asset (bonds) and one risky asset (equities). Bonds return a real risk-free rate, $r_b > 0$, while the real return on equities is normally distributed with mean r_e and variance σ^2 . Each period t , there is a generation born (which consists of just one individual) and each individual maximizes his utility, which is described by a log utility function $U(C) = \ln C + \ln C'$, subject to a lifetime budget constraint.⁴ In maximizing utility, each individual must choose current and future consumption, C and C' , and an allocation

rule, ω , which is a rule governing the optimal portfolio allocation of wealth between the risk-free and risky asset. As a consequence of utility maximization, each individual must decide how much to save, S , which will be a function of asset returns, ω , taxes, and lifetime income. In what follows, we assume that savings is a fixed amount and does not vary systematically with the above-mentioned variables. This is primarily to simplify the analysis and for the intuition. In the Appendix, a much more fully specified model is described. The results in this section represent a lower bound; making more realistic additions only enhances the results, but much of the intuition is lost.

As Merton (1969) shows, each individual would optimally like to allocate a fraction of his wealth, ω , to equities and $1-\omega$ of savings, S , to holding bonds where

$$\omega = \frac{r_e - r_b}{\sigma^2} \quad (1)$$

Let F denote the real value of the Social Security Trust Fund, *i.e.*, this is the amount of wealth that the Social Security Administration has control over and must decide how to allocate between the two assets. We let υ be the fraction that the trust fund allocates to the equity market. The parameter υ is set by policy and is therefore not a function of risk. Currently, $\upsilon=0$ which could be regarded as an infinitely risk-averse investor. A policy change in the value of υ is therefore observationally equivalent to a change in preference for a very large investor. This is in contrast to individual investors who change their allocation, ω , as a result of changes in asset returns, r_e and r_b .

We assume that there is a fixed quantity of debt and capital, D and K respectively, in the next period, and changes in the return of a given asset are determined by the current values. Therefore, $D/(1+r_b)$ is the market value of government debt that must be marketed to either Social Security or individuals. Also by assumption, there is general agreement (*i.e.*,

⁴ The notation of a prime, ', after a variable denotes the value in the next period.

homogeneous expectations) on the expected future values of bonds and stocks at time 2 based on the expected growth in the real assets of the economy, which means that the expected-future values are constants. Conceptually, one can think of bonds being paid off at face value at maturity and the real assets of each firm being liquidated to pay off the stockholders with a known expected value.⁵ The total amount of funds available to be invested in either market is $S + F$.

The market clearing conditions for the bond and equity market are respectively given by

$$(1 - \omega)S + (1 - \nu)F = \frac{D'}{(1 + r_b)} \quad (2)$$

$$\omega S + \nu F = \frac{K'}{(1 + r_e)} \quad (3)$$

For simplicity, the above equations assume a closed economy, omitting foreign investment and capital flows. Although we recognize foreign investment in domestic assets, the evidence suggests that foreign capital flows do not equate real interest rates internationally (Mishkin [1984], Throop [1994], and Fujihara and Mougoue [1996]).

Combining the market clearing conditions in equations (2) and (3) with the optimal portfolio rule, equation (1), results in a system of three equations in three unknowns, r_b , r_e , and ω .

Using the Implicit Function Theorem and Cramer's Rule, the partial derivatives $\frac{\partial r_b}{\partial \nu}$ and $\frac{\partial r_e}{\partial \nu}$ are

$$\frac{\partial r_b}{\partial \nu} = \frac{K'F(1 + r_b)^2 \sigma^2}{K'S(1 + r_b)^2 + SD'(1 + r_e)^2 + D'K'\sigma^2} > 0 \quad (4)$$

⁵ Note that this model has the characteristic that the value of future assets is not a function of the current allocation decisions so that no wealth will be created simply by reallocating funds to the equities market.

$$\frac{\partial r_e}{\partial \nu} = -\frac{D'F(1+r_e)^2\sigma^2}{K'S(1+r_b)^2 + SD'(1+r_e)^2 + D'K'\sigma^2} < 0 \quad (5)$$

These results show that r_e will decrease and r_b will increase following a shift of Social Security Trust Fund money into the equities market. To give some intuition to these equations, one could conceptually think of the chain of events following reform as follows: (1) Social Security reallocates some of the Trust Fund to the equities market, (2) since there is more money in the equity market and less in the bond market, equity prices rise and bond prices fall, causing equity returns to fall and bond returns to rise (direct effects), (3) individuals reallocate their portfolios in response to the changing returns, which offsets some of the aforementioned effects, and, (4) the previous steps (2) and (3) are repeated as individuals continue to reallocate their portfolios until a new equilibrium is achieved (indirect effects).⁶

3. Changing the Size of the Social Security Trust Fund

The model in the previous section examines the effect on returns of shifting Social Security funds to the equities market. This section uses a very similar setup to examine the effect on returns of changing the size of the Social Security Trust Fund. After finding a relationship between these two situations, one can then examine a range of possible historical relationships to estimate the potential effects of reform. Historically none of the Trust Fund money has ever been invested in the equities market. Setting $\nu=0$ in equations (2) and (3) and making similar assumptions as were made in Section 2 yields

$$(1 - \omega)S + F = \frac{D'}{(1 + r_b)} \quad (6)$$

⁶ We obviously recognize that within a general equilibrium model these events occur simultaneously, but it will be convenient later on to think of this chain of events.

$$\omega S = \frac{K'}{(1+r_e)} \quad (7)$$

Social Security is a dedicated buyer of Treasury securities, therefore, only the amount $\frac{D'}{1+r_b} - F$ of government debt must be marketed to the public. Once again, this is a system of three equations and the partials with respect to a change in F are

$$\frac{\partial r_b}{\partial F} = -\frac{K'\omega(1+r_b)^2\sigma^2}{K'S(1+r_b)^2 + SD'(1+r_e)^2 + D'K'\sigma^2} < 0 \quad (8)$$

$$\frac{\partial r_e}{\partial F} = \frac{D'\omega(1+r_e)^2\sigma^2}{K'S(1+r_b)^2 + SD'(1+r_e)^2 + D'K'\sigma^2} < 0 \quad (9)$$

In response to an increase in the size of Social Security, there are two effects. First, as Social Security increases in size, savings decrease by the same amount (since the total funds available are fixed). Therefore, immediately following the increase in Social Security there are now more funds in the bond market which causes the bond prices to rise and bond returns to fall (a qualitatively opposite effect occurs in the equity market). As a result of this initial direct effect, individuals reallocate their portfolios in response to the changing relative returns (they move the funds they have control over out of the bond market and into the equity market) and this process continues until a new equilibrium is achieved. Therefore, as the size of Social Security rises, bond returns fall and equity returns increase.

4. Adjustment Factor

It is important to note that an increase in υ in the first model has a qualitatively similar effect on returns as a decrease in the size of Social Security in the second model, but not

quantitatively the same effect. An adjustment factor can be determined, which would make the two regimes equivalent or comparable in terms of the effect on returns, by finding the ratio of partial derivatives

$$\frac{\frac{\partial r_b}{\partial \nu}}{\frac{\partial r_b}{\partial F}} = -\frac{F}{\omega} \quad (10)$$

or

$$\frac{\partial r_b}{\partial \nu} = \left(-\frac{F}{\omega}\right) \frac{\partial r_b}{\partial F} \quad (11)$$

Recalling that the two above regimes only differed in terms of their direct effects, the resulting effect on returns will be the same whether Social Security reallocates z percent of the Trust Fund to the equities market or whether the size of the Social Security Trust Funds falls by $z \frac{F}{\omega}$. Therefore, a \$1 shift of the Trust Fund into the equities market has more of an effect on interest rates than a \$1 decrease in the size of the Trust Fund (equivalently, a \$1 shift of the Trust Fund into the equities market has the same effect as a $\$ \frac{1}{\omega} > 1$ decrease in the size of the Trust Fund). For example, if the Trust Fund shifts \$1 into the equities market, the full \$1 goes to the equities market, and then the returns change as individuals alter their portfolios accordingly. If the Trust Fund were to decrease by \$1 (giving this \$1 back to individuals), individuals would only shift $\$ \omega$ to the equities market (and $\$(1-\omega)$ to the bond market). Then the returns change, and individuals alter their portfolios accordingly. As shown above, the secondary effects are the same, but these two reforms have different primary effects (initial movement of funds), which the adjustment factor addresses.

5. Results and Implications

In this section, we empirically relate a change in interest rates from increasing the size of the Social Security Trust Fund to a potential shifting in the investment of these funds to include equities. We use the following data: size of the Social Security Trust Fund, bond and equity market capitalization, 3-month T-Bill rates, 10-year T-Bond rates, Aaa corporate bond rates, GDP, monetary base, price level, inflation rate, expected inflation, and predicted measures of the size of the Trust Fund, publicly held federal government debt, wage income, and inflation. The historical data are annual from 1952 to 1996.

As mentioned above, our objective is to compare the effects of the two different reforms on interest rates. To accomplish this, we need an estimate of the relationship between the size of the Social Security Trust Fund and interest rates.⁷ Since the Social Security Trust Fund is held in the form of Treasury Securities, an increase in the Trust Fund of \$1 reduces the amount of debt the federal government must market to the public by \$1. Therefore, the effect on interest rates of an increase in the Trust Fund is exactly equal to a \$1 decrease in publicly held government debt.

The research addressing the effects of government deficits or the level of debt on interest rates is well established (Motley [1983], Hoelscher [1983], Barth *et. al.* [1984], DeLeeuw and Holloway [1985], Evans [1985,1987], Darrat [1989], CBO[1987], etc.), but without any consensus with respect to either the direction or magnitude of the effects. These studies differ in a number of ways. Various authors have attempted to estimate these effects using interest rates on different maturing government securities, different data frequencies, and different econometric techniques. Some have allowed for time-varying effects or have modeled expectations of future deficits while others have incorporated an international sector, considered monetary policy, and/or examined the direction of causality. Since the main contribution of this paper is the

adjustment factor mentioned above, and not the resolution of the estimation of the effects of government policy on interest rates, we use a range of specifications to estimate this relationship that results in a range of possible effects of the two reforms. We consider six different dependent variables: 3-month T-Bill rates (ex-post and ex-ante), 10-year T-Bond rates (ex-post and ex-ante), and Aaa corporate bond rates (ex-post and ex-ante). The ex-post rates are determined by subtracting the realized inflation rate from the appropriate nominal rates and the ex-ante rates are found by subtracting the expected inflation rate (as measured using the Livingston Survey data). The regressors included government debt, monetary base, unemployment rate, equity market capitalization, expected inflation, equity market return, one-period lagged interest rate.⁸

The results are reported in Table 1. Economic theory predicts that government debt is capable of having a negative, positive, or no relationship on interest rates. The results presented in Table 1 suggest that the potential estimates range from not significant to a significant estimate of 0.0021; the significant estimates range from 0.0012 to 0.0021. This is interpreted to mean that for every billion dollar increase in publicly held debt, interest rates increase by zero (as insignificant) to 2.1 basis points.⁹ Our position is that a positive relationship between government debt and interest rates is intuitive. Therefore, from a policy initiative point of view, we believe that the range of possibilities must be considered even though we do not attempt to resolve different opinions on the relative size of any relationship between government debt and interest rates.

⁷ An important point to note is that empirically we are only examining the effect of Social Security reform on interest rates although the above models predict that reform should also affect equity returns in a qualitatively opposite manner.

⁸ The nominal variables were deflated three different ways: by the price level, by nominal GDP, and as the natural log of the real measure. Various combinations of these variables were examined, but the other results are entirely consistent with those reported here.

⁹ This is the coefficient on the most inclusive regression using *ex-post* real interest rates on 10-year T-securities.

5.1 Effect on Interest Rates of Investing Social Security Funds in Equities

The first type of reform we examine is a shift of Social Security Funds to the equities market. Unfortunately, we do not have any direct historical evidence to indicate the effect of such a movement of funds. The models developed in Sections 2 and 3 suggest it is possible to use the historical relationship between the size of the Social Security Trust Fund and interest rates along with the adjustment factor to estimate the effect of reform on returns. As shown in Section 4, there is an equivalence between the effects on interest rates of a reallocation of Trust Fund money and a change in the size of the Trust Fund. In other words, interest rates would rise from a shift of Social Security funds to the equity market or a decrease in the size of Social Security. However, as noted, the equivalence is not one-for-one and the adjustment factor is necessary to make the two actions comparable. That adjustment is given by $\frac{F}{\omega}$. For example, a z percent shift of Social Security funds to the equities market will raise interest rates by the same as if the size of Social Security fell by $z \frac{F}{\omega}$.

We use our estimates, along with the current CBO projections of the size of the Trust Fund, to estimate the effect of this type of reform on interest rates over the next ten years.¹⁰ The size of the effect on interest rates from a reform in which Social Security Trust funds are shifted to the equities market obviously will depend on the amount of funds shifted to the equities market (note the SS term in the adjustment factor) and ω . We use CBO estimates (which are very close to those from the Social Security Administration) of the projected size of the Trust Fund and a practical value of 0.7 for ω . The resulting effect on interest rates is found by using the relationship in equation (31) and is reported in Table 2. These numbers are interpreted as follows: if the reform were to take place in 1999, interest rates would increase by (0,102) basis points at that time relative to their current level. If the reform waited until 2008, interest rates would

increase by (0,216) basis points relative to their current level.¹¹ More generally, the reform, if it happens, will occur over many years but presumably will be complete by 2008. Therefore, instead of interest rates falling by (0,216) basis points in the absence of any reform (due to projected budget surpluses over the next ten years), we predict they will be (0,216) basis points higher.

5.2 The Effect on Income Taxes of Investing Social Security Funds in Equities

Some of the Social Security Reform proposals suggest that it is possible to avoid an increase in the Social Security tax rate by simply shifting Trust Fund money into the equities market. This section demonstrates that although it may appear that a tax increase would be avoided, general income taxes will increase following the movement of investments into equities.

The period-by-period governmental budget constraint is given as

$$D' = (1 + r_b)D + g - T \quad (12)$$

where D' is the future level of government debt, D is the current level of government debt, g is current government expenditures, T is the current income-tax. Suppose that the government sets τ in order to keep its debt constant, *i.e.*, $D' = D = \bar{D}$. Therefore, T can be expressed as

$$T = r_b \bar{D} + g \quad (13)$$

Therefore, as Social Security Trust Funds are shifted to the equities market and r_b increases,

obviously, income taxes must increase as well by $\frac{\partial r_b}{\partial v} \bar{D}$. The intuition behind this result is that

¹⁰ Note that these estimates do not explicitly consider the growth in the Trust Fund due to the higher returns on the portion invested in the equities market. }

¹¹ Note that the increase in interest rates will be much greater relative to their future levels since they will decrease over time due to the increase in the size of the Trust Fund as mentioned above.

when Social Security funds are shifted into the equities market, bond returns increase as the government must now entice more investors to buy its debt. In other words, the interest portion of government spending increases, so income taxes need to be increased to maintain a constant debt level. The increase in interest rates has substantial implications for the federal government and fiscal policy. If the government maintains a stable fiscal position by holding total federal debt constant, this increase in interest costs will necessitate an increase in other tax revenues. As the interest portion of government spending increases (while other expenditures remain constant), there must be an increase in income taxes to make-up the shortfall of revenues now relative to expenditures; with a potential rise in interest rates, the cost of servicing the total federal debt of \$5.6 trillion in 2000 would increase. Alternatively, Social Security will benefit from this movement of funds to the equities market in two ways: (1) Higher interest rates will increase the return on funds invested in the bond market, and (2) Funds moved to the equities market will have a higher expected return than the bond market.

Since the total government debt is substantially larger than the Trust Fund, the cost to the federal government are magnified compared to the benefits that accrue to Social Security because higher interest payments are made to all bondholders, not just Social Security. In addition to higher income taxes, the net effects of shifting Social Security funds into equities is also passed on in the form of lower equity returns (implications to be discussed below). There are no free lunches, that is, the Social Security system cannot become fully-funded through the simple movement of funds to the equities market without an increase in the total tax burden. Notice that under such a scenario, an increase in the Social Security tax is avoided, but there is a necessary increase in income taxes.

6. Other Implications of Social Security Investment in Equities

The potential implications for income redistribution resulting from reform are of particular interest, and must be compared to some benchmark policy; we take the benchmark case

as the current system in which distant future benefits are cut. In the absence of reform, the Social Security Trust Fund is not forecast to be depleted for three decades (at which time benefits will have to be reduced by 25%). Under the benchmark case, there is no effect on current equity returns, interest rates, or income taxes. If there is a single representative agent then any reform will be neutral from the standpoint that the costs to this agent of are offset by the benefits. A more interesting case occurs when there are heterogeneous individuals differentiated by whether they are U.S. citizens or not, where they derive their income (capital, bonds, or labor), and their age.

An Alternative to decreasing future benefits is to increase current payroll taxes. As the above model (and the more complete model in the Appendix) demonstrates, this reform results in equity returns decreasing, interest rates increasing, and income taxes decreasing (since the interest expense portion of government spending decreases). Relative to the benchmark case, under this reform equityholders benefit, bondholders are hurt, taxpayers benefit, current workers are hurt, and future recipients are not hurt as bad as under the benchmark case. Here, there is an intergenerational income transfer from today's workers to tomorrow's recipients (the degree of overlap between these two groups depends on how far into the future "tomorrow" is considered to be). Furthermore, there is an income transfer from current workers to taxpayers since the increase in the payroll tax decreases interest rates allowing income taxes to be reduced.

Another method to maintain a solvency in the Social Security system is to shift Social Security Trust Funds to the equities market. As the model above suggests, this reform causes equity returns to decrease, interest rates to increase, and necessitates an increase in income taxes to maintain a constant fiscal stance. Clearly under this reform, equityholders are hurt, bondholders benefit, taxpayers are hurt, current workers are left unaffected (to the extent that under the benchmark their payroll taxes were not affected as they are under the "shifting funds" reform), and future recipients benefit (to the extent that under their benefits are left unchanged compared to being cut under the benchmark case). Under this reform, there is an income transfer

from taxpayers and equityholders to future recipients (and current workers compared to the “increasing payroll tax” reform).

To the extent that all bonds and equities are domestically held, the choice of reform is determined by which groups (equityholder, bondholders, current workers, future Social Security recipients) the government favors. If all groups are weighted equally, then the government should be indifferent between the reforms (the problem would be reduced to having one infinitely-lived representative agent from the government’s point of view). If all bonds and equities are not domestically held then there will be income transfers between domestic residents and foreigners. For example, under the “increase payroll taxes” reform, there is a redistribution from U.S. workers (paying higher payroll taxes) to foreigners holding U.S. equities since the return on U.S. equities increases following this reform. Furthermore, following this type of reform, there would be a transfer of income from foreign bondholders (holding bonds paying a lower interest rate) to U.S. taxpayers (paying lower income taxes because the interest expense portion of government spending is reduced).

Alternatively, under the “shift funds” approach there is an income redistribution from foreigners holding U.S. equities to workers. In this case, foreigners holding U.S. equities would lose because they are earning a lower return on their equity holdings but not receiving any benefits from the Social Security system, effectively subsidizing U.S. workers’ retirements. The effects of this type of reform is further complicated by the fact that a substantial portion of the U.S. Government’s debt (approximately one third of publicly held debt) is held by foreigners. This creates a political dilemma in which there is a redistribution of income from U.S. taxpayers (paying higher income taxes) to foreigners holding US bonds (paying higher interest rates).

According to the latest CBO budget, the government is expecting to run total surpluses in the amount of \$5.6 trillion over the next decade. The surpluses are divided between Social Security (\$2.5 trillion) and the rest of the government (\$3.1 trillion). Therefore, the arguments presented above concerning increased interest rates, increased interest payments, and tax

increases could be modified in light of these projected surpluses. As stated above, the necessary tax increases are for a constant fiscal stance. If the government wants to continue to run a \$3.1 trillion surplus over the next decade then the above arguments still hold. These tax increases are relative to the government's fiscal position. Clearly, the government could absorb higher interest payments and not increase taxes, but the resulting surpluses would decrease by the amount of the increased interest payments. Alternatively, the tax cut that the government could offer in light of the surpluses, would partially be offset by higher interest payments. On net, higher interest payments may not seem as painful in an environment of budget surpluses, but only to an irrational individual who does not see that higher interest payments cause taxes to be cut by a smaller amount than they otherwise would be.

The shifting of Trust funds from Treasury securities to equities has generally been sold to the public as a painless cure for the salvation of Social Security, but we have shown that this is not so straightforward. In fact, shifting funds to the equities market could actually be a more painful solution to the problem (from the standpoint of American taxpayers) if there is an income redistribution to foreign bondholders. The change in the *total* tax burden under these alternative plans should be an integral part of any discussion of Social Security Reform.

7. Summary and Conclusion

Several proposals have been developed to reform the Social Security System in order to ensure that it is fully funded. These proposals include investing a portion of Social Security funds in equities instead of increasing Social Security payroll taxes. A general equilibrium model is developed in this research to demonstrate some of the effects of these reform proposals. The model shows that investing a portion of Social Security funds in equities will increase interest rates on bonds. We demonstrate that shifting funds into the equities market is equivalent to a

change in the size of the Social Security Trust Fund times an adjustment factor. Using historical data, we show a significant relationship between interest rates and the size of the Social Security Trust Fund. From this empirical relationship, we then demonstrate that interest rates could increase if a portion of the existing Trust Fund is invested in equities. Furthermore, while the Trust Fund will have a higher expected return, other taxes may need to be increased to maintain an equivalent budget position. An interesting extension of this model is to examine the alternative reform proposal of increasing the payroll tax. Intuitively, if the payroll tax is increased, this would increase the size of the Trust Fund, decrease the amount of debt the government must market to the public, lower interest rates, and allow for a reduction in taxes. The stark contrast with investing a portion of the Trust Fund in the equities market stresses the importance of this issue when considering alternative proposals.

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Table 1
Regression Results

	rrb10-p	rrb10-a	rrb3-p	rrb3-a	aaa-p	aaa-a
Debt _{1,1}	0.00188 (0.02)	0.00150 (0.01)	0.00183 (0.02)	0.00116 (0.02)	0.00170 (0.02)	0.00130 (0.01)
Debt _{1,2}	0.00208 (0.02)	0.00064 (0.25)	0.00199 (0.02)	0.00063 (0.33)	0.00198 (0.02)	0.00035 (0.51)
Debt _{1,3}	0.00145 (0.08)	0.00060 (0.28)	0.00131 (0.08)	0.00050 (0.41)	0.00140 (0.09)	0.00033 (0.54)
Debt _{2,1}	0.034 (0.30)	0.046 (0.03)	0.033 (0.28)	0.033 (0.12)	0.035 (0.29)	0.048 (0.03)
Debt _{2,2}	0.034 (0.32)	0.036 (0.08)	0.039 (0.21)	0.021 (0.34)	0.032 (0.33)	0.40 (0.05)
Debt _{2,3}	4.041 (0.17)	0.037 (0.08)	0.024 (0.35)	0.024 (0.26)	0.042 (0.18)	0.041 (0.05)
Debt _{3,1}	3.80 (0.01)	2.82 (0.01)	3.94 (0.01)	2.50 (0.01)	3.30 (0.02)	2.36 (0.01)
Debt _{3,2}	3.89 (0.01)	1.84 (0.05)	4.27 (0.01)	1.75 (0.10)	3.45 (0.02)	1.33 (0.14)
Debt _{3,3}	3.07 (0.03)	1.87 (0.05)	2.76 (0.02)	1.81 (0.02)	2.62 (0.05)	1.33 (0.14)

Debt_{i,j} refers to the coefficient on the government debt variable. i refers to the specification of independent variables: i=1, constant, government debt, monetary base, unemployment, GDP, equity market capitalization, and one-period lagged interest rate; i=2, constant, government debt, monetary base, unemployment, GDP, equity market capitalization, expected inflation, and one-period lagged interest rate; i=3, constant, government debt, monetary base, unemployment, GDP, equity market capitalization, expected inflation, equity market return, and one-period lagged interest rate. j refers to the deflating method of the independent variables: j=1, independent variables are measured in real terms; j=2, independent variables measured as relative to GDP; j=3, independent variables measured as natural logarithm of real values. The dependent variables are rrb10= 10-year T-bond, rrb3 = 3-month T-bill, aaa = Aaa corporate bond. -p=*ex-post*, -a=*ex-ante*. P-values are in parentheses. Bold numbers are significant at the 10% level.

Table 2
Change in Interest Rates:

Shifting Funds (Basis Points: Using Upper Bound Estimates)

Year	Δr
1999	120
2000	114
2001	126
2002	138
2003	152
2004	164
2005	177
2006	190
2007	205
2008	216

Appendix

A.1 Individual Behavior

Each period, there is a representative individual born who lives for two periods. The individual maximizes his utility that is described by a log utility function

$$U = \ln C_t + \ln C_{t+1} \quad (14)$$

In addition, the individual is endowed with an initial wealth level of W_t . Furthermore, each individual must pay lump-sum income taxes, T_t , and lump-sum Social Security tax, T_t^S , in the first period of his life. In the second period of life, the individual receives Social Security benefits, B_{t+1} . Each individual must decide how much to consume in each period as well as how to allocate his savings between the risky and risk-free asset. Therefore, the lifetime budget constraint of an individual born in period t is given by

$$W_t - T_t - T_t^S + \frac{B_{t+1}}{R_t} = C_t + \frac{C_t}{R_t} \quad (15)$$

where $R_t = \omega_t(1+r_{e,t}) + (1-\omega_t)(1+r_{b,t})$. As Merton (1969) shows, an individual with this utility function would optimally like to allocate a fraction of their wealth, ω_t , to equities and $1-\omega_t$ of their wealth, W_t , to holding bonds where

$$\omega_t = \frac{r_{e,t} - r_{b,t}}{\sigma^2} \quad (16)$$

The optimal period t consumption and savings, S_t , rules for an individual born in period t are given by

$$C_t = \frac{1}{2} \left[W - T_t - T_t^S + \frac{B_{t+1}}{R_t} \right] \quad (17)$$

$$S_t = \frac{1}{2} \left[W - T_t - T_t^S - \frac{B_{t+1}}{R_t} \right] \quad (18)$$

The savings of generation t is used to buy the existing government debt and capital stock, K_t , from the previous generation as well as to finance any new investment in that may occur due to changing asset returns (this will be discussed below).

A.2 Social Security

Let X_t denote the real value of the Social Security Trust Fund entering into period t . In each period t , the Social Security Trust Fund grows by the difference in Social Security taxes collected and benefits paid, $T_t^S - B_t$. Therefore, the amount of funds that Social Security has to allocate and invest between the two assets is $F_t = X_t + T_t^S - B_t$. Let υ denote the fraction that the trust fund allocates to the equity market. The parameter υ is set by policy and is therefore not a function of risk. Currently, $\upsilon=0$ which could be regarded as an infinitely risk-averse investor. A policy change in the value of υ is therefore observationally equivalent to a change in preference for a very large investor. This is in contrast to individual investors who change their allocation, ω_t as a result of changes in asset returns, $r_{e,t}$ and $r_{b,t}$.

A.3 Government Budget Constraint

Each period, the non-Social Security portion of the government collects taxes, T_t and has some constant level of government spending of g . We assume that at period t the government has a stock of outstanding debt that has a face value of D_t . The evolution of this government debt is given by

$$D_{t+1} = (1 + r_{b,t})D_t + g - T_t \quad (19)$$

We assume that the government sets the tax rate, T_t , in order to keep the par value of debt constant at D . Therefore, taxes are set to finance government spending and the interest on the existing debt, as follows:

$$T_t = r_b D + g \quad (20)$$

A.4 Evolution of the Capital Stock

Below, we examine the impact of shifting Social Security Funds to the equities market as well as increasing the size of the Trust Fund. These actions could potentially cause asset returns and savings to change. As savings changes, if the level of government debt is constant and the capital stock is constant then changing savings must simply result in changing asset prices. We allow for the possibility that firms will engage in investment activities that increase the capital stock the following period. Therefore, we assume that investment, I_t , is a negative function of $r_{e,t}$

$$I_t = \frac{d}{(1 + r_{e,t})} \quad (21)$$

where d is a parameter determining the sensitivity of investment to changes in the equity returns.

A.5 Market Clearing Conditions

We assume that the debt, D_t , the capital stock, K_t , and investment, I_t , are financed each period by the individual's current savings, S_t , and Social Security. The market clearing conditions for the bond and equity market are respectively given by

$$(1 - \omega_t)S_t + (1 - \nu)(X_t + T_t^S - B_t) = \frac{D}{1 + r_{b,t}} \quad (22)$$

$$\omega_t S_t + \nu(X_t + T_t^S - B_t) = K_t + \frac{d}{1 + r_{e,t}} \quad (23)$$

A.6 Solution

Therefore, the economy is described by the following equations that describe savings, taxes, bond market-clearing condition, equity market-clearing condition, optimal portfolio choice, and a weighted return:

$$S_t = \frac{1}{2} \left[W - T_t - T_t^S - \frac{B_{t+1}}{R_t} \right] \quad (24)$$

$$T_t = r_b D + g \quad (25)$$

$$(1 - \omega_t) S_t + (1 - \nu)(X_t + T_t^S - B_t) = \frac{D}{1 + r_{b,t}} \quad (26)$$

$$\omega_t S_t + \nu(X_t + T_t^S - B_t) = K_t + \frac{d}{1 + r_{e,t}} \quad (27)$$

$$\omega_t = \frac{r_{e,t} - r_{b,t}}{\sigma^2} \quad (28)$$

$$R_t = \omega_t(1 + r_{e,t}) + (1 - \omega_t)(1 + r_{b,t}) \quad (29)$$

From these equations, it is possible to solve for the impact on asset returns as the Trust Fund grows. Substituting for R_t and T_t into (24), (26), (27), and (29) results in four equations in four unknowns, $r_{b,t}$, $r_{e,t}$, ω_t , and S_t as functions of X_t , ν , W_t , B_t , B_{t+1} , d , K_t , and g . Define the functions as follows:

$$\frac{1}{2} \left[W_t - r_{b,t} D - g - T_t^S - \frac{B_{t+1}}{R_t} \right] - S_t = 0 \equiv M^1 \quad (30)$$

$$(1 - \omega_t) S_t + (1 - \nu)(X_t + T_t^S - B_t) - \frac{D}{1 + r_{b,t}} = 0 \equiv M^2 \quad (31)$$

$$\omega_t S_t + \nu(X_t + T_t^S - B_t) - K_t - \frac{d}{1 + r_{e,t}} = 0 \equiv M^3 \quad (32)$$

$$\frac{r_{e,t} - r_{b,t}}{\sigma^2} - \omega_t = 0 \equiv M^4 \quad (33)$$

The time subscripts are omitted when there is no resulting confusion. Using the implicit function theorem and Cramer's Rule, the impact on interest rates of a change in the size of the Trust Fund is found by taking the total differentials of the above four equations and isolating the effect of a change in T^S by setting all other differentials equal to zero. Note that the Trust Fund grows because T^S increases while keeping benefits constant. The resulting system can be rewritten as

$$\begin{bmatrix} \frac{\partial M^1}{\partial M^2} & \frac{\partial M^1}{\partial M^3} & \frac{\partial M^1}{\partial M^4} & \frac{\partial M^1}{\partial \omega} \\ \frac{dr_b}{\partial M^2} & \frac{dr_e}{\partial M^3} & \frac{dS}{\partial M^4} & \frac{d\omega}{} \\ \frac{\partial M^2}{\partial M^2} & \frac{\partial M^2}{\partial M^3} & \frac{\partial M^2}{\partial M^4} & \frac{\partial M^2}{\partial \omega} \\ \frac{dr_b}{\partial M^3} & \frac{dr_e}{\partial M^3} & \frac{dS}{\partial M^3} & \frac{d\omega}{} \\ \frac{\partial M^3}{\partial M^2} & \frac{\partial M^3}{\partial M^3} & \frac{\partial M^3}{\partial M^4} & \frac{\partial M^3}{\partial \omega} \\ \frac{dr_b}{\partial M^4} & \frac{dr_e}{\partial M^4} & \frac{dS}{\partial M^4} & \frac{d\omega}{} \\ \frac{\partial M^4}{\partial M^2} & \frac{\partial M^4}{\partial M^3} & \frac{\partial M^4}{\partial M^4} & \frac{\partial M^4}{\partial \omega} \\ \frac{dr_b}{\partial M^4} & \frac{dr_e}{\partial M^4} & \frac{dS}{\partial M^4} & \frac{d\omega}{} \end{bmatrix} \begin{bmatrix} \left(\frac{\partial r_b}{\partial T^S} \right) \\ \left(\frac{\partial r_e}{\partial T^S} \right) \\ \left(\frac{\partial S}{\partial T^S} \right) \\ \left(\frac{\partial \omega}{\partial T^S} \right) \end{bmatrix} = \begin{bmatrix} -\frac{\partial M^1}{\partial T^S} \\ -\frac{\partial M^2}{\partial T^S} \\ -\frac{\partial M^3}{\partial T^S} \\ -\frac{\partial M^4}{\partial T^S} \end{bmatrix} \quad (34)$$

By Cramer's Rule, the partial $\left(\frac{\partial r_b}{\partial T^S} \right) = \frac{|\mathbf{J}_{r_b}|}{|\mathbf{J}|}$ where \mathbf{J}_{r_b} is the matrix that is formed by replacing

the first row above with the vector on the right-hand-side of the equation and \mathbf{J} is the matrix on the left-hand-side of the equation.

$$\mathbf{J} = \begin{bmatrix} \frac{1}{2} \left(-D + \frac{B(1-\omega)}{R^2} \right) & \frac{B\omega}{2R^2} & -1 & \frac{b(r_e - r_b)}{2R^2} \\ \frac{D}{(1+r_b)^2} & 0 & 1-\omega & -S \\ 0 & \frac{d}{(1+r_e)^2} & \omega & S \\ -\frac{1}{\sigma^2} & \frac{1}{\sigma^2} & 0 & -1 \end{bmatrix}$$

Note that the column vector on the right-hand-side of the equation above is

$$\begin{bmatrix} -\frac{\partial M^1}{\partial T^S} \\ -\frac{\partial M^2}{\partial T^S} \\ -\frac{\partial M^3}{\partial T^S} \\ -\frac{\partial M^4}{\partial T^S} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad (36)$$

$$|\mathbf{J}_{r_b}| = .5 \begin{vmatrix} 0 & \omega & -S \\ \frac{d}{(1+r_e)^2} & 1-\omega & S \\ \frac{1}{\sigma^2} & 0 & -1 \end{vmatrix} + (-1) \begin{vmatrix} \frac{B\omega}{2R^2} & -1 & \frac{B(r_e - r_b)}{2R^2} \\ 0 & \omega & -S \\ \frac{1}{\sigma^2} & 0 & -1 \end{vmatrix}$$

Therefore, $\left| \mathbf{J}_{r_b} \right|$ is found as

$$\begin{aligned}
&= .5 \left[\omega S \left(\frac{1}{\sigma^2} \right) - \left(\frac{1}{\sigma^2} \right) (1 - \omega) (-S) - (-1) \left(\frac{d}{(1 + r_e)} \right) \omega \right] \\
&- 1 \left[\left(\frac{B\omega}{2R^2} \right) \omega (-1) + (-1) (-S) \left(\frac{1}{\sigma^2} \right) - \left(\frac{1}{\sigma^2} \right) \omega \left(\frac{B(r_e - r_b)}{2R^2} \right) \right] \quad (38) \\
&= \frac{d\omega\sigma^2 R^2 - SR^2(1 + r_e)^2 + 2B\omega(1 + r_e)(r_e - r_b)}{2\sigma^2 R^2(1 + r_e)^2} < 0
\end{aligned}$$

It can be shown that $|\mathbf{J}| > 0$ and $|\mathbf{J}_{r_b}| < 0$ so interest rates fall as the Trust Fund increases which is qualitatively similar to the results in the paper. Furthermore, it can be shown that individual savings increases as the Trust Fund increases.

In order to find the impact of a shift of Social Security Funds, replace the first column of $|\mathbf{J}|$ with the following matrix (defining F as the total number of funds that Social Security has to allocate to bonds and equities) which is $|\mathbf{J}_v|$

$$\begin{bmatrix} -\frac{\partial M^1}{\partial v} \\ -\frac{\partial M^2}{\partial v} \\ -\frac{\partial M^3}{\partial v} \\ -\frac{\partial M^4}{\partial v} \end{bmatrix} = \begin{bmatrix} 0 \\ (X + T^S - B) \\ -(X + T^S - B) \\ 0 \end{bmatrix} = \begin{vmatrix} 0 \\ F \\ -F \\ 0 \end{vmatrix} \quad (39)$$

$$|\mathbf{J}_v| = -F \begin{vmatrix} \frac{B\omega}{2R^2} & -1 & \frac{B(r_e - r_b)}{2R^2} \\ \frac{d}{(1+r_e)^2} & 1-\omega & S \\ \frac{1}{\sigma^2} & 0 & -1 \end{vmatrix} + (-F) \begin{vmatrix} \frac{B\omega}{2R^2} & -1 & \frac{B(r_e - r_b)}{2R^2} \\ 0 & \omega & -S \\ \frac{1}{\sigma^2} & 0 & -1 \end{vmatrix}$$

This results in $|\mathbf{J}_v|$ being found as

$$\begin{aligned} &= -F \left[\frac{B\omega}{2R^2} (1-\omega)(-1) + (-1)S \left(\frac{1}{\sigma^2} \right) - \left(\frac{1}{\sigma^2} \right) (1-\omega) \left(\frac{B(r_e - r_b)}{2R^2} \right) - (-1)(-1) \left(\frac{d}{(1+r_e)^2} \right) \right] \\ &- F \left[\frac{B\omega}{2R^2} \omega(-1) + (-1)(-S) \left(\frac{1}{\sigma^2} \right) - \left(\frac{1}{\sigma^2} \right) \omega \left(\frac{B(r_e - r_b)}{2R^2} \right) \right] \\ &= \frac{2FB(1+r_e)^2(r_e - r_b) + 2d\sigma^2 R^2 F}{2\sigma^2 R^2 (1+r_e)^2} > 0 \end{aligned}$$

Therefore, the partial derivative $\frac{\partial r_b}{\partial T^s}$ is found by $\frac{|\mathbf{J}_{r_b}|}{|\mathbf{J}|}$ and the partial $\frac{\partial r_b}{\partial \nu}$ is found by $\frac{|\mathbf{J}_\nu|}{|\mathbf{J}|}$ and

therefore the ratio of $\frac{\frac{\partial r_b}{\partial \nu}}{\frac{\partial r_b}{\partial T^s}}$ is found by the ratio $\frac{|\mathbf{J}_\nu|}{|\mathbf{J}_{r_b}|}$. This ratio is

$$\frac{\frac{2FB(1+r_e)^2(r_e-r_b)+2d\sigma^2R^2F}{2\sigma^2R^2(1+r_e)^2}}{\frac{d\omega\sigma^2R^2-SR^2(1+r_e)^2+2B\omega(1+r_e)(r_e-r_b)}{2\sigma^2R^2(1+r_e)^2}} = \frac{2FB(1+r_e)^2(r_e-r_b)+2d\sigma^2R^2F}{d\omega\sigma^2R^2-SR^2(1+r_e)^2+2B\omega(1+r_e)(r_e-r_b)}$$

which simplifies to

$$= \frac{2FB(1+r_e)^2(r_e-r_b)+2d\sigma^2R^2F}{d\omega\sigma^2R^2-SR^2(1+r_e)^2+2B\omega(1+r_e)(r_e-r_b)} \quad (43)$$

This adjustment factor will be smaller (greater in absolute value) than the one used in the paper if

$$\frac{2FB(1+r_e)^2(r_e-r_b)+2d\sigma^2R^2F}{d\omega\sigma^2R^2-SR^2(1+r_e)^2+2B\omega(1+r_e)(r_e-r_b)} < -\frac{F}{\omega} \quad (44)$$

Since the denominator is negative, this is equivalent to showing that

$$2B(1+r_e)^2(r_e-r_b)\omega+2d\sigma^2R^2\omega > d\omega\sigma^2R^2-SR^2(1+r_e)^2+2B\omega(1+r_e)(r_e-r_b)$$

which is clearly true since the right-hand-side of the equation is negative while the right-hand-side is positive. Therefore, the adjustment factor used in the paper is a lower bound and within a more realistic model, the adjustment factor would be greater.

It is also possible to show that individual savings decreases as the Trust Fund shifts funds to the equities market which is opposite to the impact on savings of increasing the size of the Trust Fund while keeping its allocation in bonds. Therefore, if we were to explicitly account for dynamic effects of these two policies, the results presented above would be exaggerated because the “increasing Trust Fund” policy would result in capital deepening, and increased future resources, as savings increases while the “shift funds to the equities market” policy would have exactly the opposite effect.